

EE 505

Lecture 6

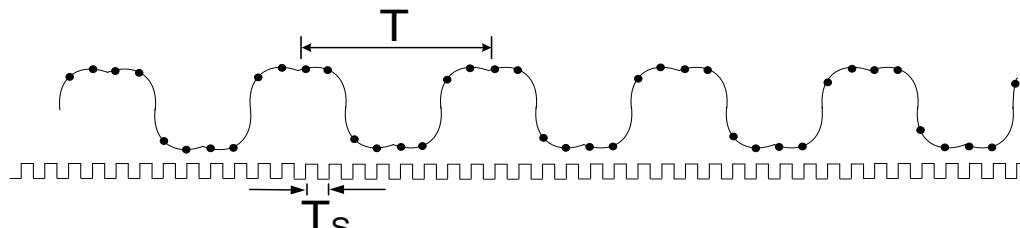
Spectral Analysis in Spectre

- Standard transient analysis
- Strobe period transient analysis

Addressing Spectral Analysis Challenges

- Problem Awareness
 - Windowing
 - Post-processing

Distortion Analysis



THEOREM: Consider a periodic signal with period $T=1/f$ and sampling period $T_s=1/f_s$. If N_p is an integer, $x(t)$ is band limited to f_{MAX} , and $f_s > 2f_{max}$, then

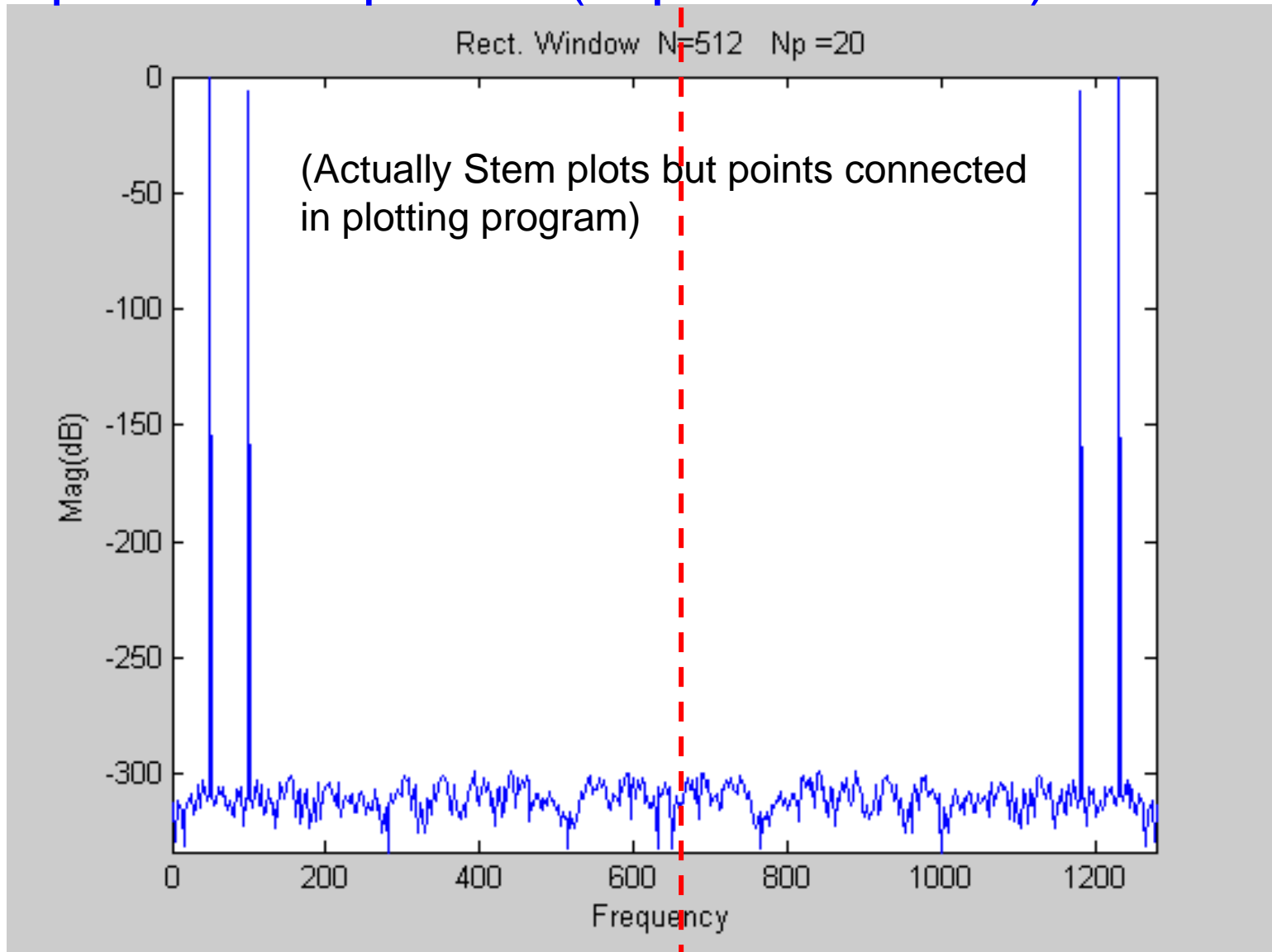
$$|A_m| = \frac{2}{N} |X(mN_p + 1)| \quad 0 \leq m \leq h - 1$$

and $X(k) = 0$ for all k not defined above

where $\langle X(k) \rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_s) \rangle_{k=0}^{N-1}$

N =number of samples, N_p is the number of periods, and $h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_p}\right)$

Spectral Response (expressed in dB)

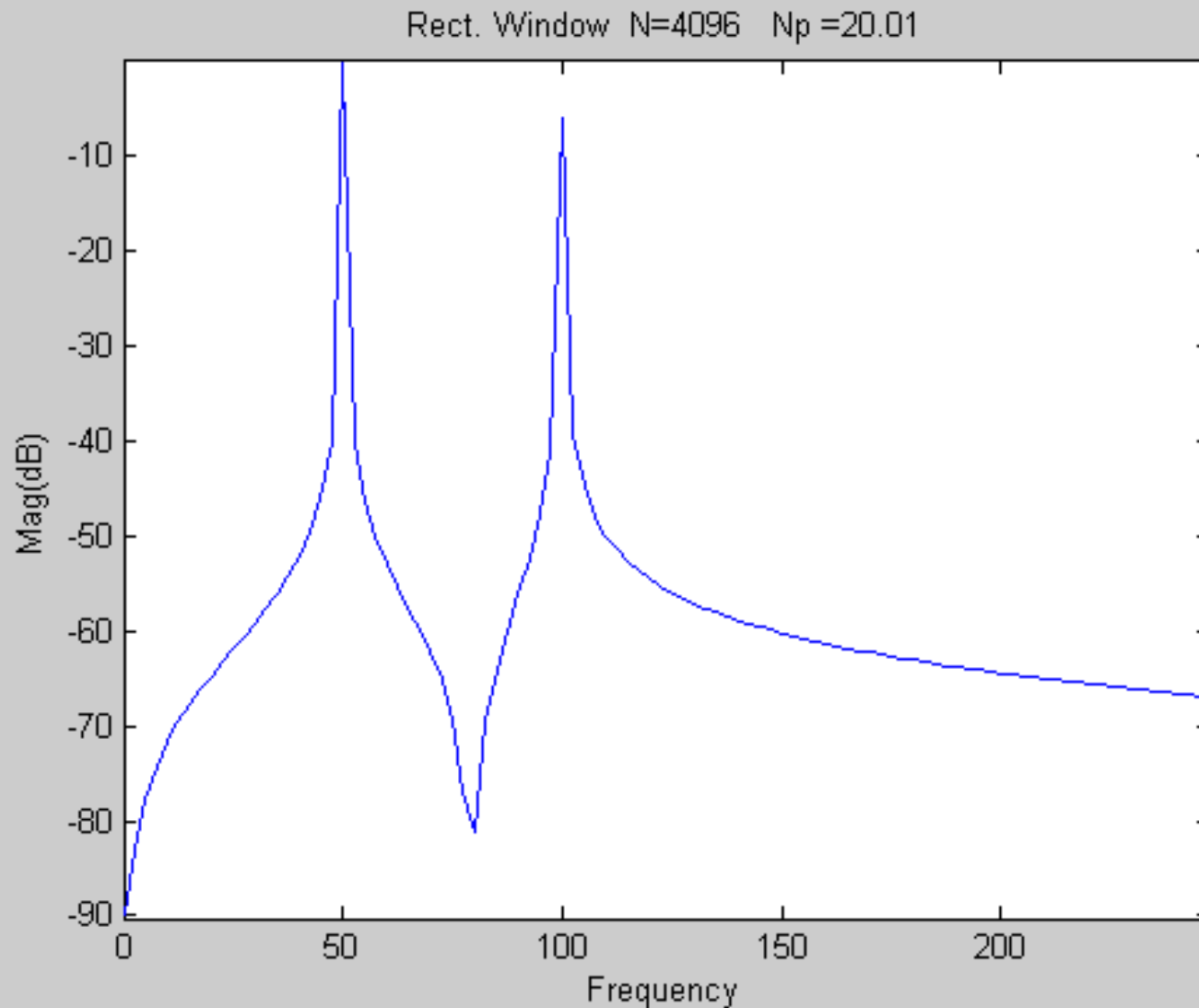


Note Magnitude is Symmetric wrt $0.5 \cdot f_{\text{SAMPLE}}$

$$f_{\text{AXIS}} = f_{\text{SIGNAL}} \frac{n-1}{N_p}$$

Spectral Response with Non-coherent Sampling

Review from last lecture



(zoomed in around fundamental)

Observations

- Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB)
- Errors at about the 13-bit level !

FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods



2.

$$N > \frac{2f_{\max}}{f_{\text{SIGNAL}}} N_P$$

Example

If $f_{\text{SIG}}=50\text{Hz}$

and $N_P=20$ $N=512$

$$f_{\text{SAMP}} = f_{\text{SIG}} \frac{N}{N_P}$$

$$f_{\text{SAMP}}=1280 \text{ Hz}$$

$$N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_P \quad \longrightarrow \quad f_{\text{max}} < 640\text{Hz}$$

Example

Consider $N_p=20$ $N=512$

If $f_{\text{SIG}}=50\text{Hz}$

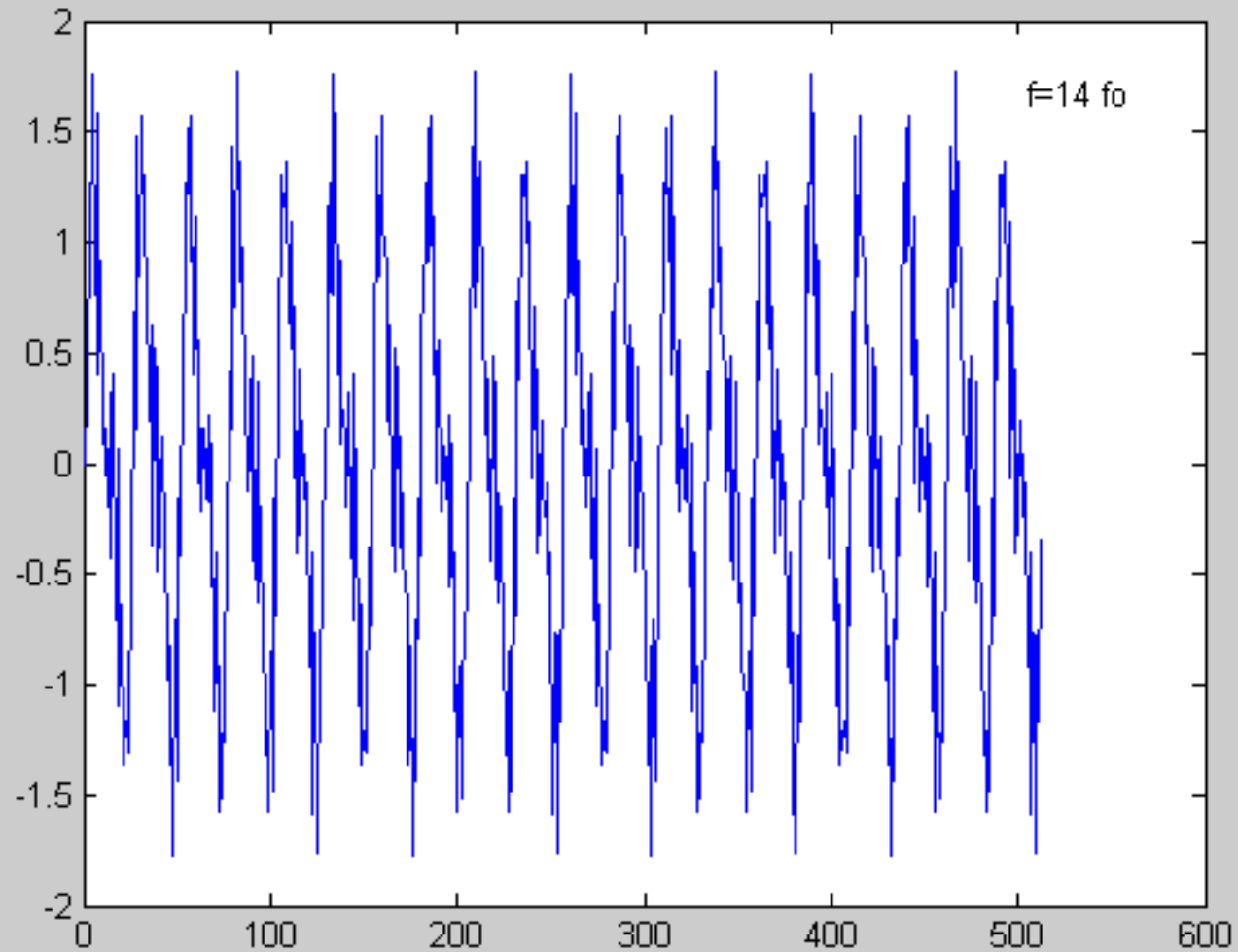
$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t) + 0.5 \sin(14\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

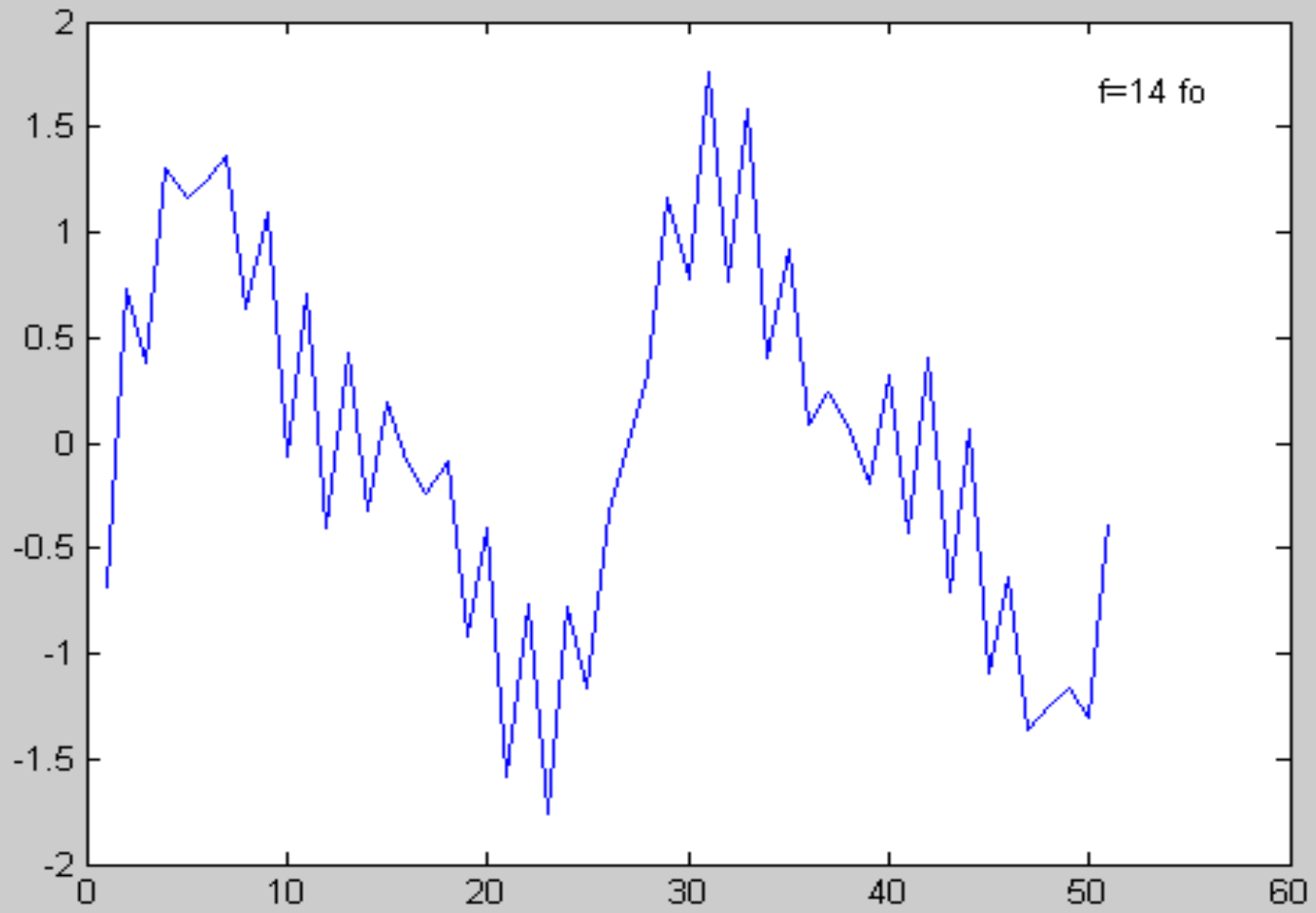
(i.e. a component at 700 Hz which violates the band limit requirement)

Recall $20\log_{10}(0.5)=-6.0205999$

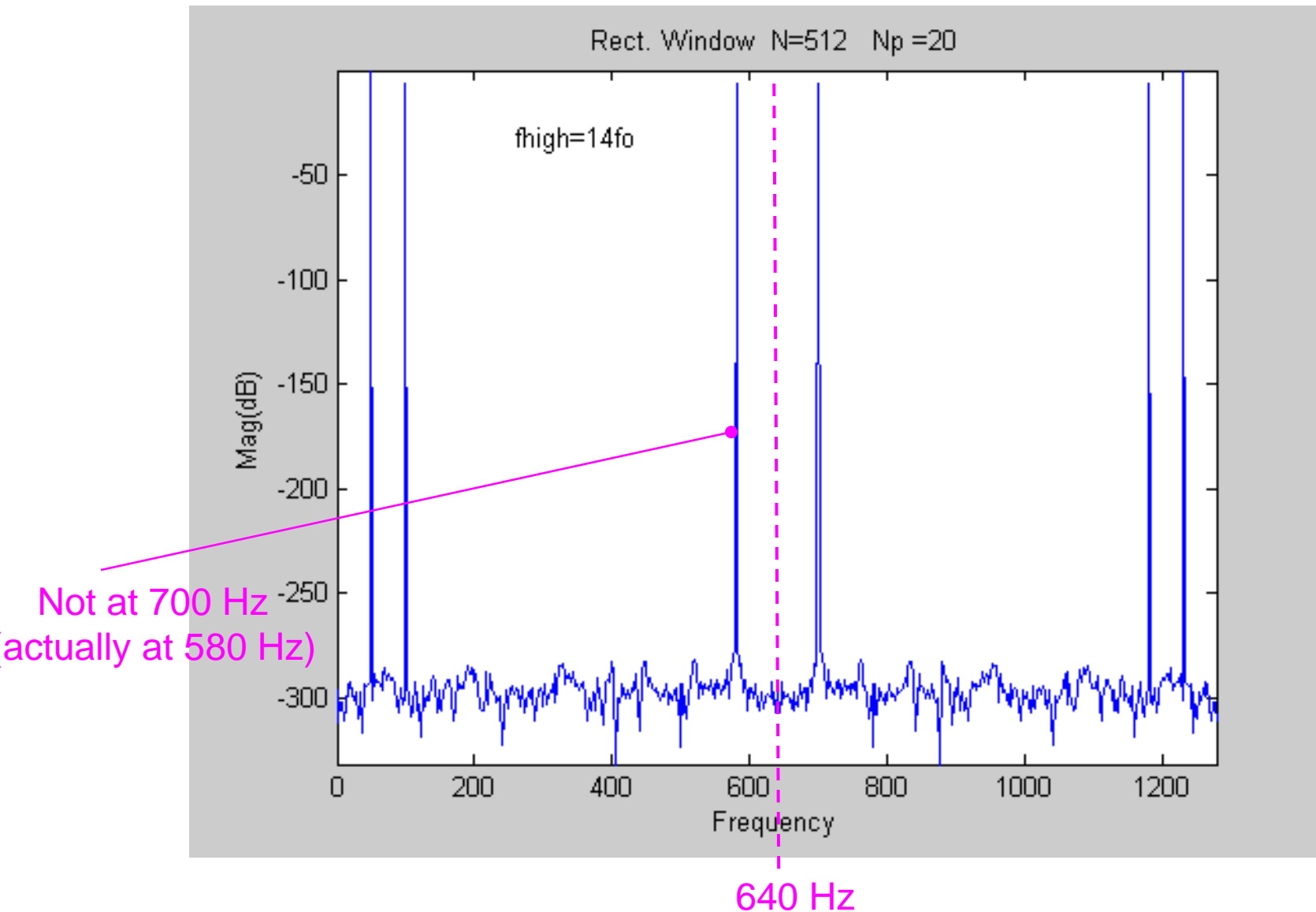
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

$$f_{\text{high}} = 14f_0$$

Columns 1 through 7

-296.9507 -311.9710 -302.4715 -302.1545 -310.8392 -304.5465 -293.9310

Columns 8 through 14

-299.0778 -292.3045 -297.0529 -301.4639 -297.3332 -309.6947 -308.2308

Columns 15 through 21

-297.3710 -316.5113 -293.5661 -294.4045 -293.6881 -292.6872 -0.0000

Columns 22 through 28

-301.6889 -288.4812 -292.5621 -292.5853 -294.1383 -296.4034 -289.5216

Columns 29 through 35

-285.9204 -292.1676 -289.0633 -292.1318 -290.6342 -293.2538 -296.8434

Effects of High-Frequency Spectral Components

$$f_{\text{high}} = 14f_0$$

Columns 36 through 42

-301.7087 -307.2119 -295.1726 -303.4403 -301.6427 -6.0206 -295.3018

Columns 43 through 49

-298.9215 -309.4829 -306.7363 -293.0808 -300.0882 -306.5530 -302.9962

Columns 50 through 56

-318.4706 -294.8956 -304.4663 -300.8919 -298.7732 -301.2474 -293.3188

Effects of High-Frequency Spectral Components

Aliased components at

$$f_{\text{alias}} = f_{\text{sample}} - f$$

$$f_{\text{alias}} = 1280 - 700 = 580\text{Hz}$$

$$\text{thus position in sequence} = 1 + N \frac{f_{\text{alias}}}{f_{\text{sample}}} = 1 + 512 \frac{580}{1280} = 233$$

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238

-273.9840 -6.0206 -274.2295 -284.4608 -283.5228 -297.6724 -291.7545

Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

Columns 246 through 252

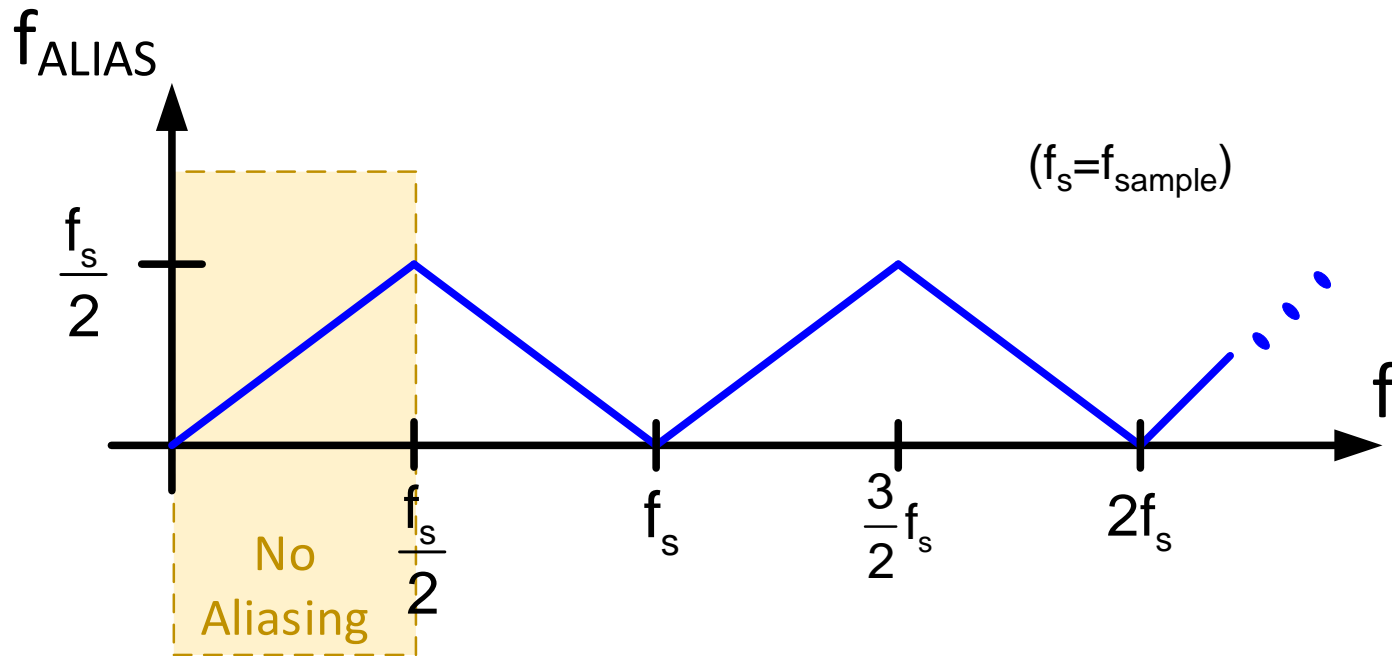
-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956

Alias frequency and Index Position

$$f_{\text{alias}} = \begin{cases} f \quad (\text{no alias}) & 0 < f < \frac{f_{\text{sample}}}{2} \\ f_{\text{sample}} - f & \frac{f_{\text{sample}}}{2} < f < f_{\text{sample}} \\ -f_{\text{sample}} + f & f_{\text{sample}} < f < \frac{3}{2}f_{\text{sample}} \\ 2f_{\text{sample}} - f & \frac{3}{2}f_{\text{sample}} < f < 2f_{\text{sample}} \\ -2f_{\text{sample}} + f & 2f_{\text{sample}} < f < \frac{5}{2}f_{\text{sample}} \\ 3f_{\text{sample}} - f & \frac{5}{2}f_{\text{sample}} < f < 3f_{\text{sample}} \quad \dots \end{cases}$$

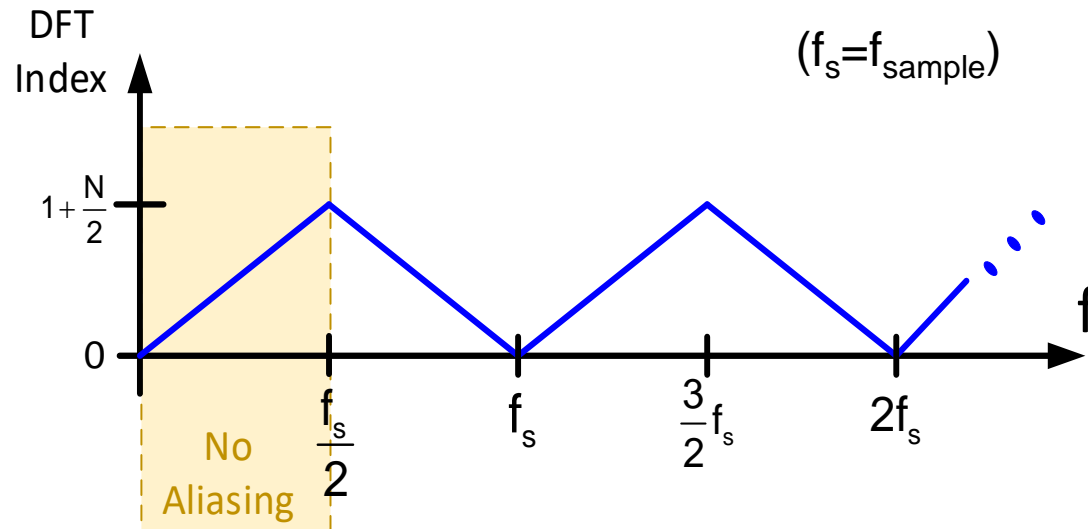
$$\text{Index position in sequence} = 1 + N \frac{f_{\text{alias}}}{f_{\text{sample}}}$$

Alias frequency and Index Position



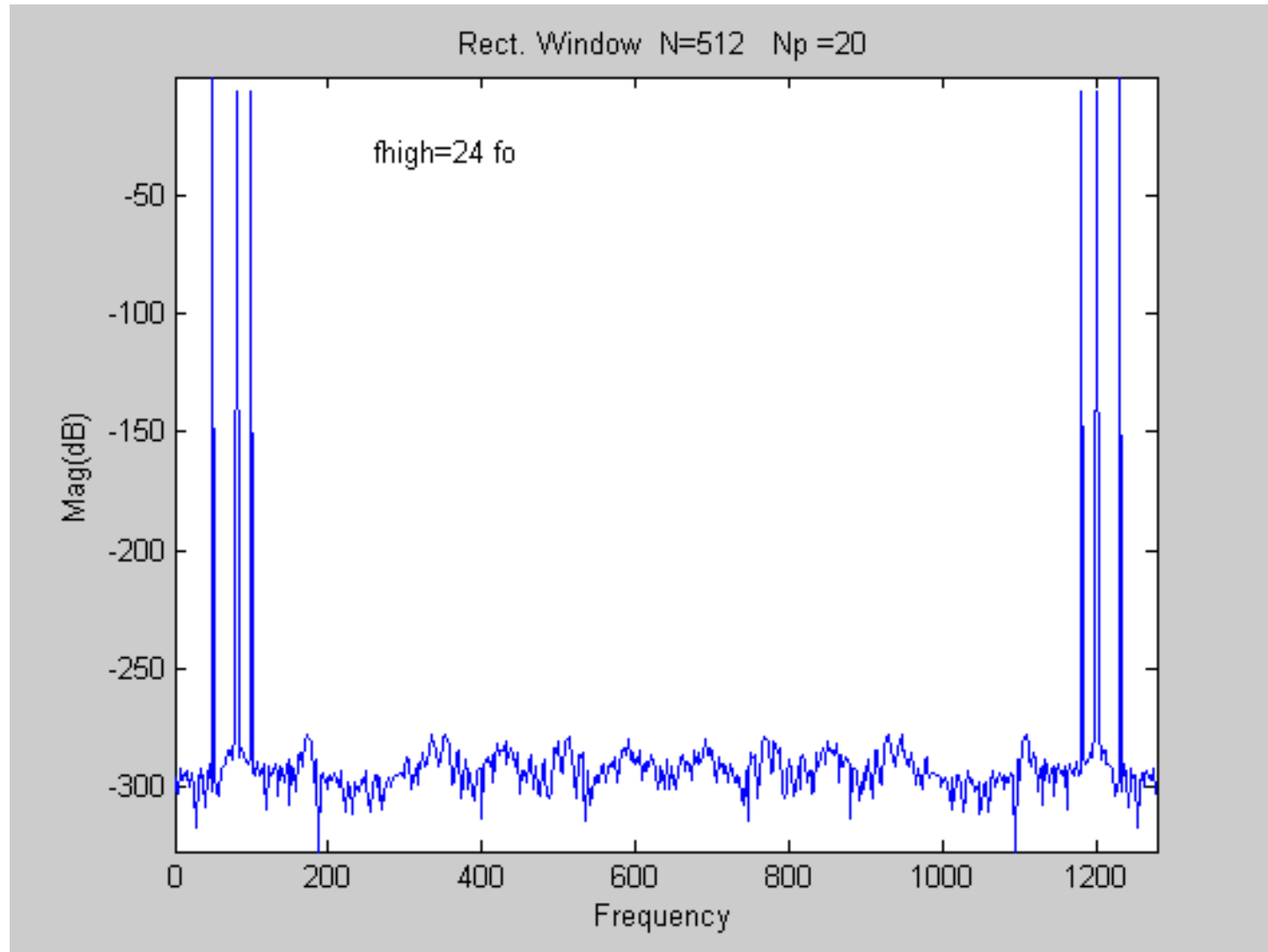
$$\text{Index position in sequence} = 1 + N \frac{f_{\text{alias}}}{f_{\text{sample}}}$$

Alias frequency and Index Position

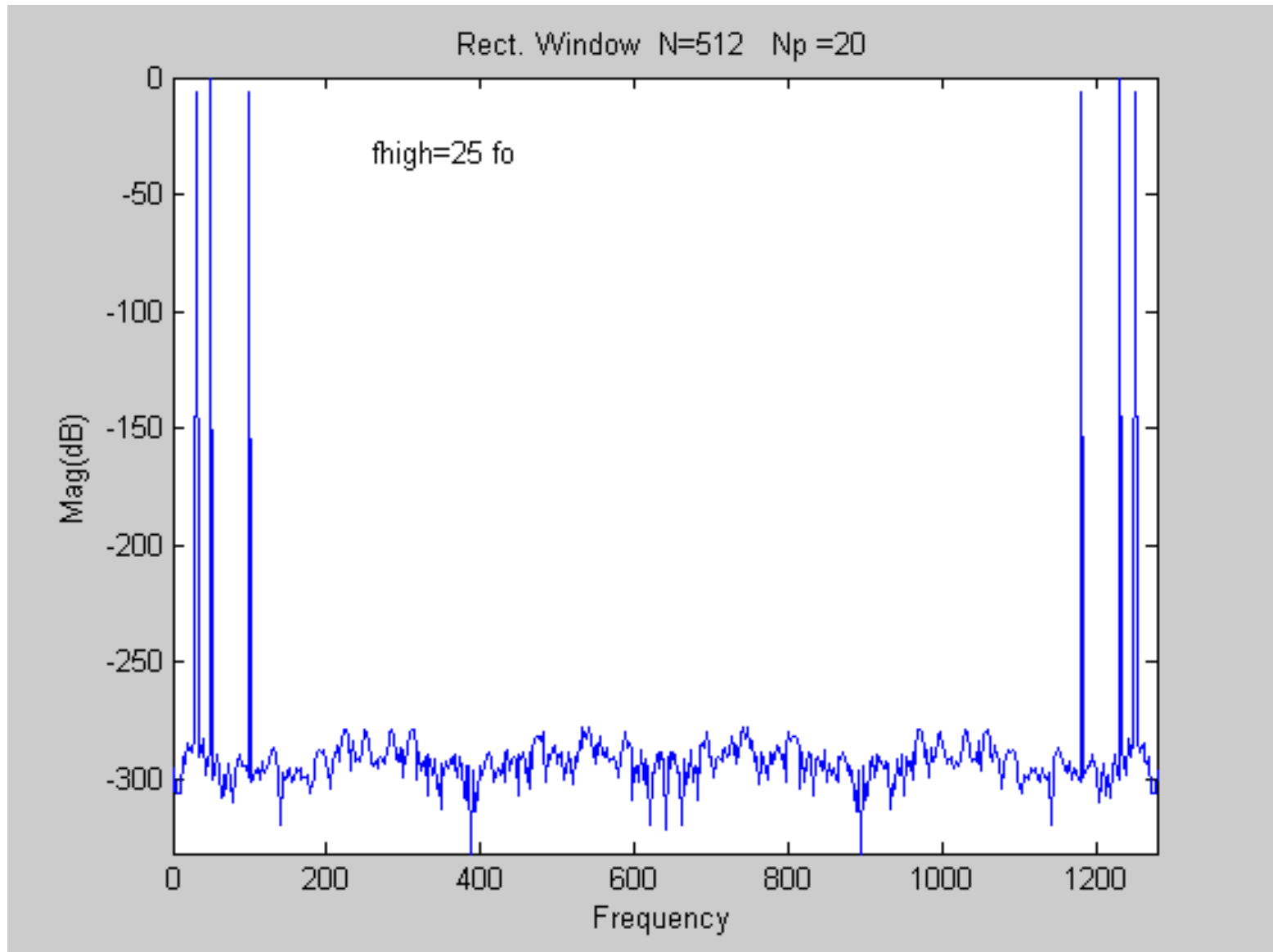


$$\text{Index position in sequence} = 1 + N \frac{f_{\text{alias}}}{f_{\text{sample}}}$$

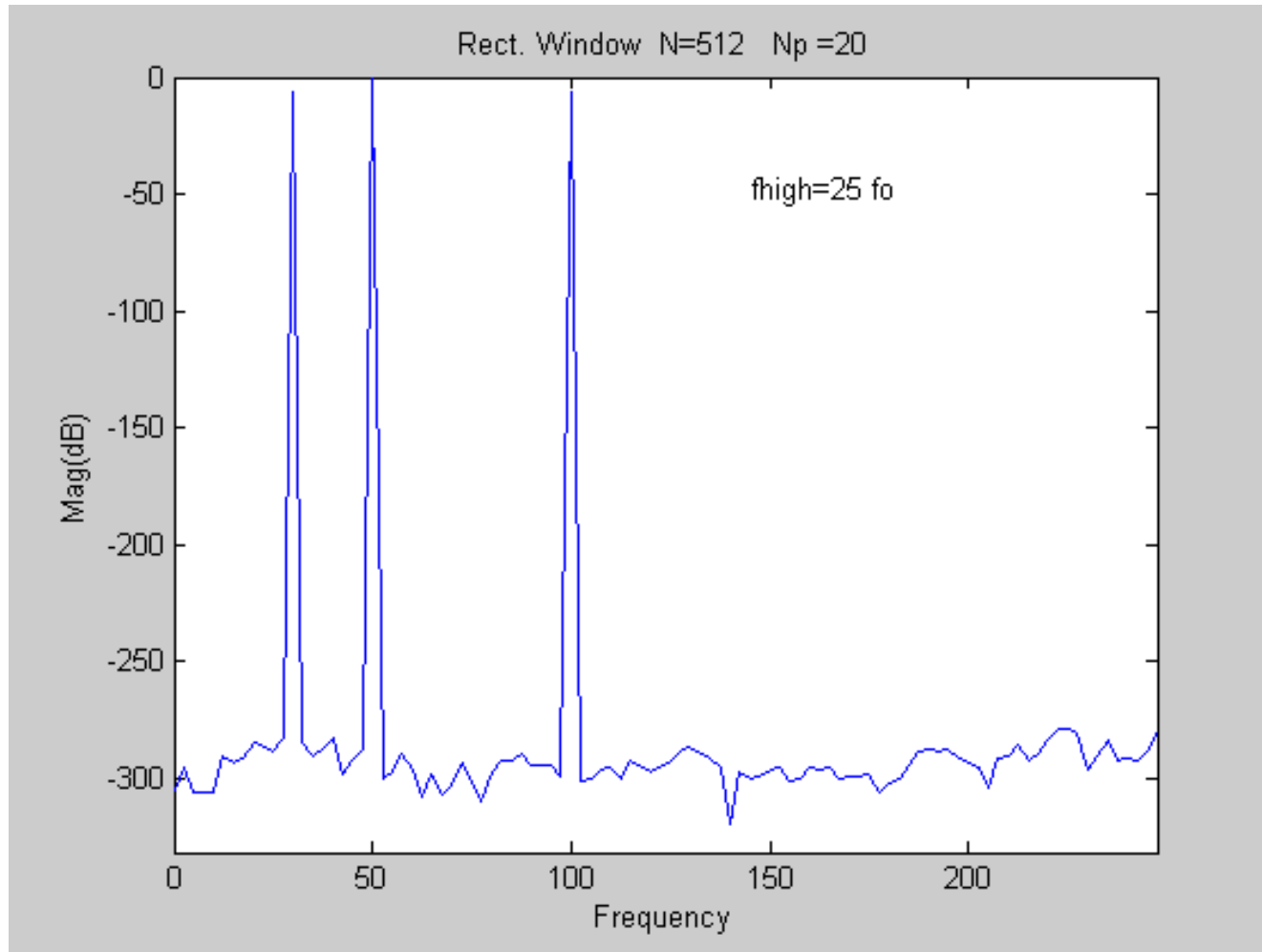
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

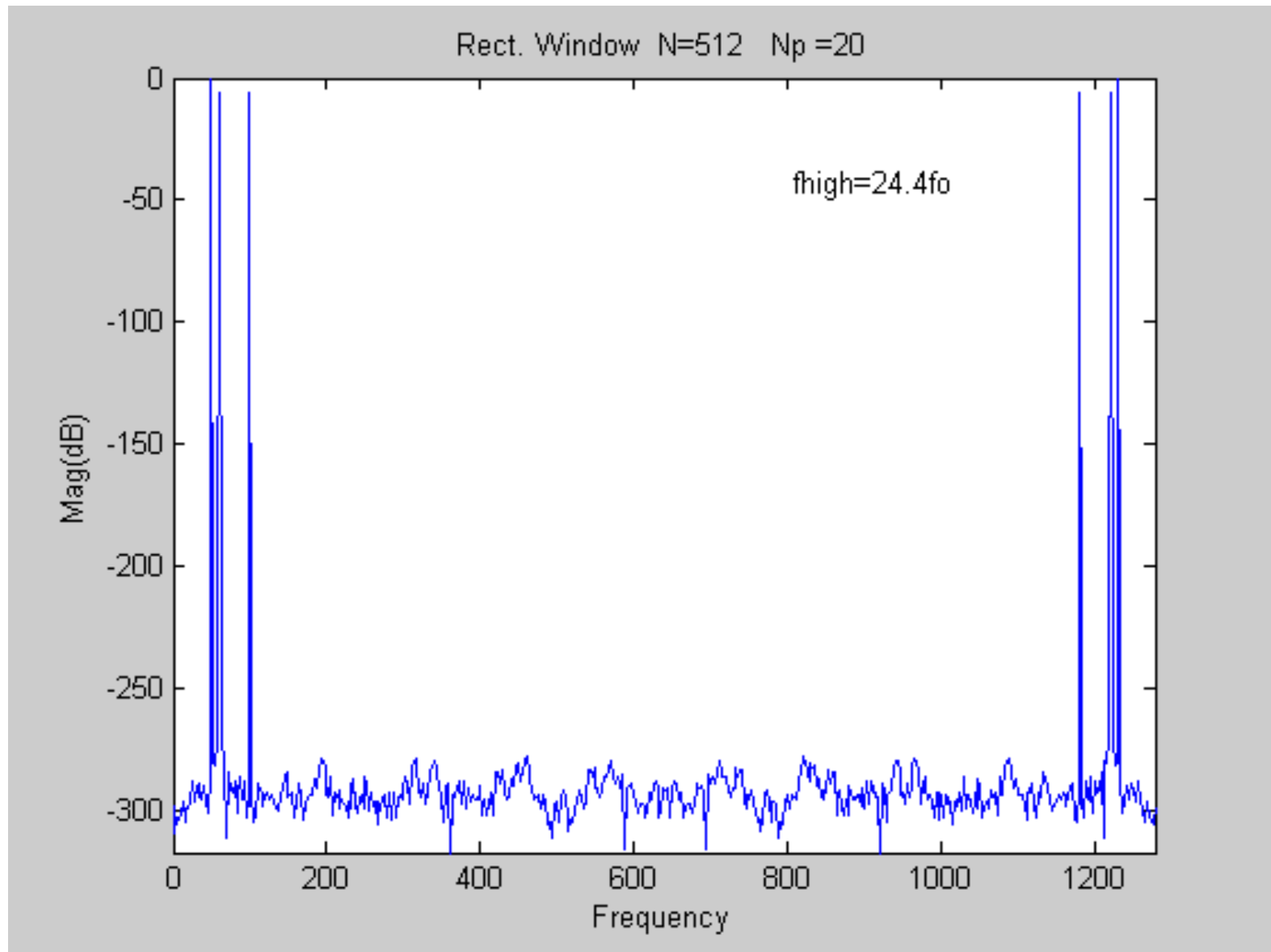


Effects of High-Frequency Spectral Components

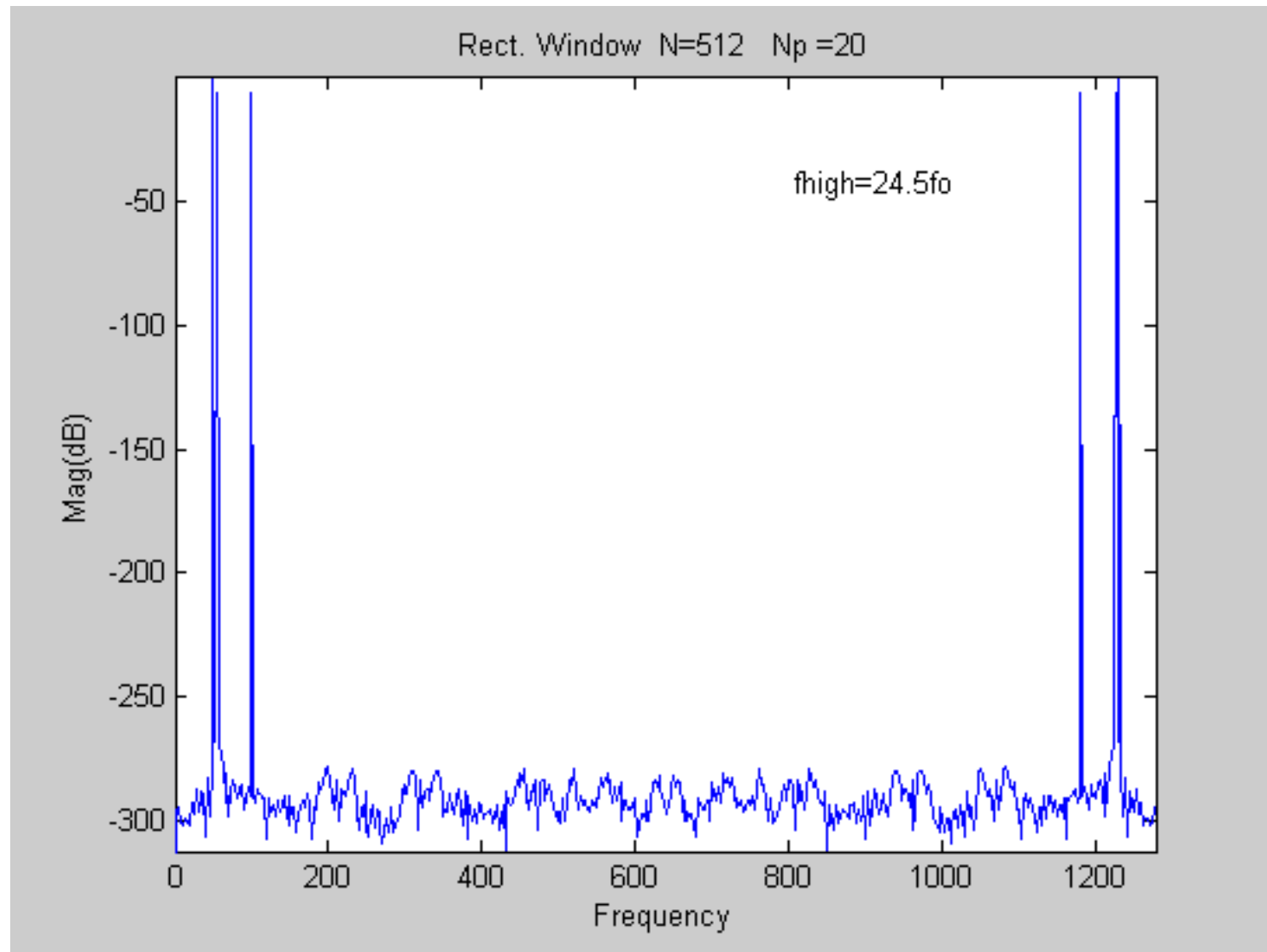


(zoomed in around fundamental)

Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

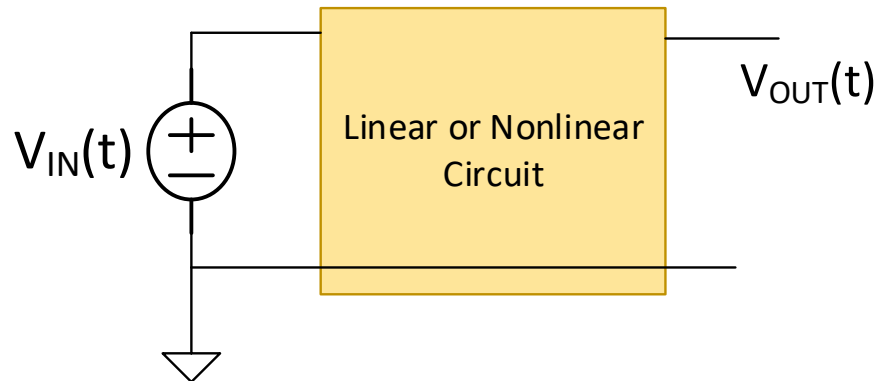


Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics
- Important to avoid aliasing if the DFT is used for spectral characterization

Quirks with Circuit Simulators

The transient simulation results obtained with almost all circuit simulators are almost always wrong!

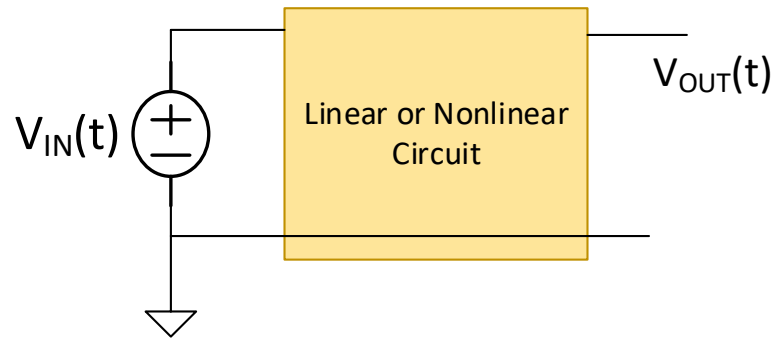


If $V_{IN} = V_m \sin(\omega t)$, the probability that any output for any linear or nonlinear circuit is the correct value is likely 0

The suppliers of all commercial circuit simulators know that their simulators almost never provide the correct solution for transient analysis in even simple circuits

But the solutions provided are often very good approximations to the actual solution

Quirks with Circuit Simulators



Obtaining an exact result for a transient simulation is an extremely challenging problem and even obtaining very good results have required major efforts by the CAD community for many decades

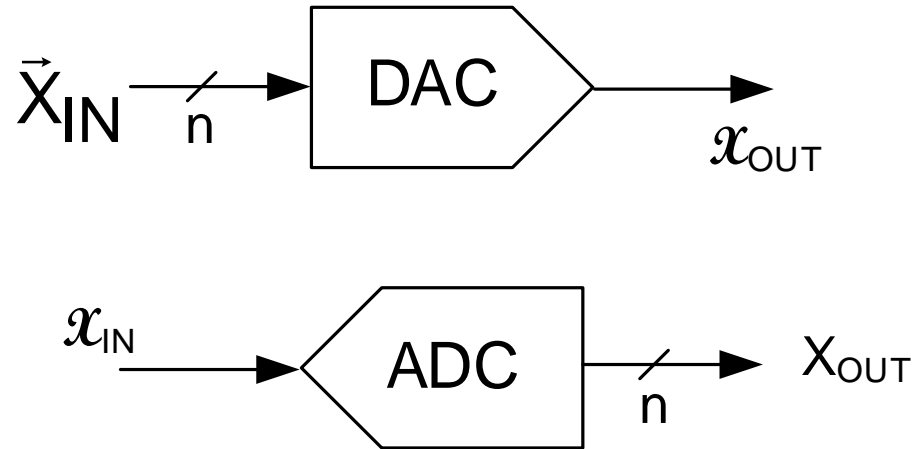
It is highly unlikely that the goal of any CAD tool vendor is to provide the exact transient solution

So what is likely the goal of a CAD tool vendor when developing tools to provide the transient response of a circuit?

Conjecture: To provide solutions that are good enough to convince customers to spend money to use the simulator !

Is this a good goal that well-serves the customer?

Quirks with Circuit Simulators

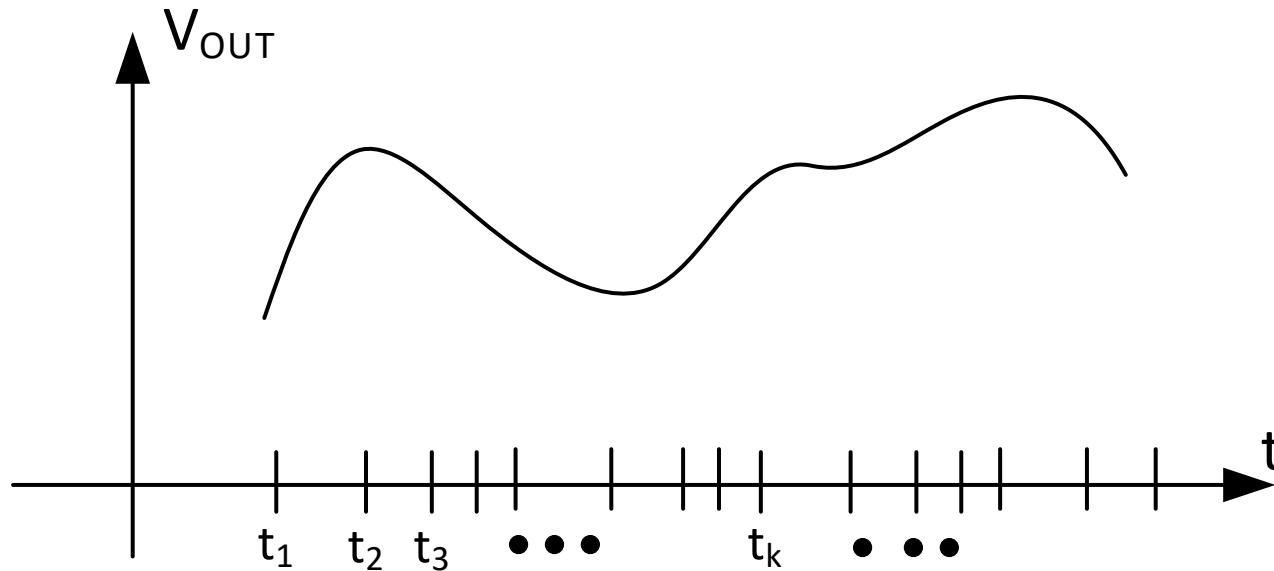


There are numerous quirks that become issues when simulating ADCs of DACs , particularly when the specifications are demanding

Will discuss one of these quirks associated with spectral characterization today

Quirks with Circuit Simulators

Time steps in transient simulations



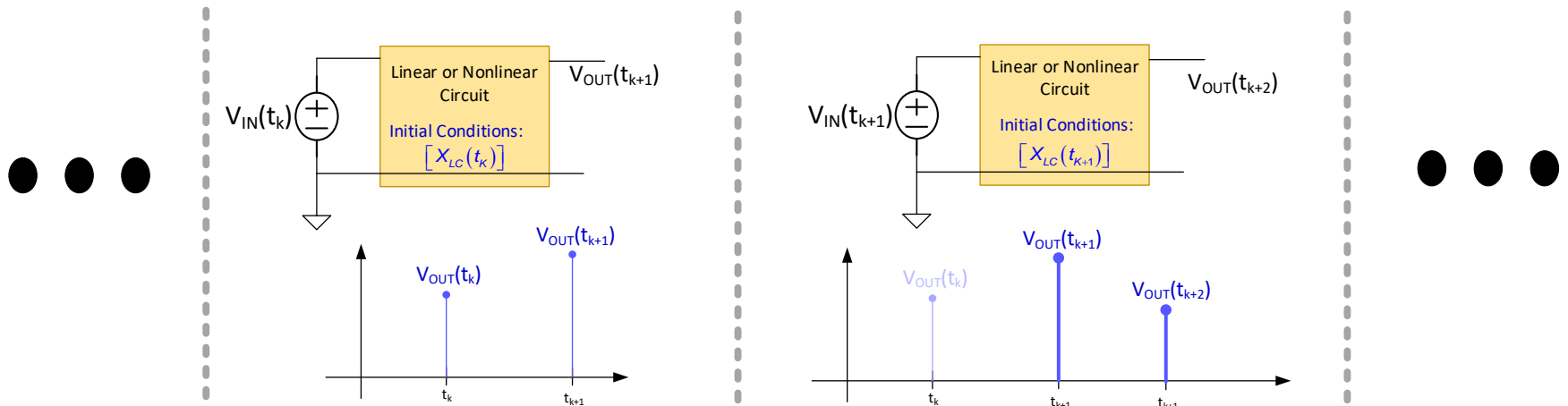
Transient analysis in SPICE involves breaking the simulation time interval into short sub-intervals of length $\hat{t}_k = t_{k+1} - t_k$

The solution involves using the calculated output at time t_k as the input at time t_{k+1}

The time steps are intentionally not uniform to provide reasonable tradeoffs between simulation time and accuracy

Quirks with Circuit Simulators

Transient Simulation Approach in Simulators

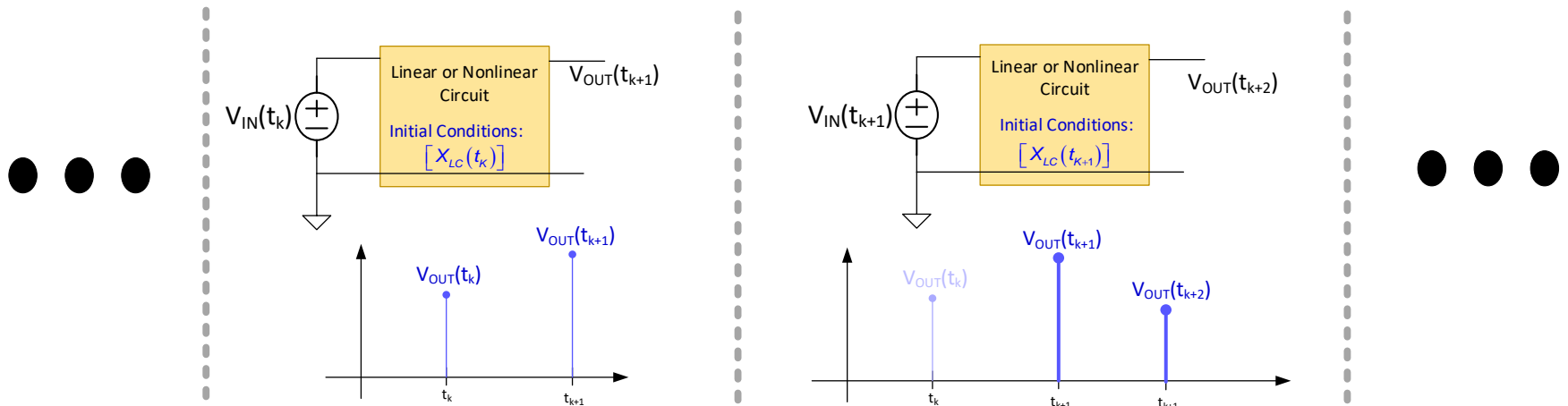


$[X_{LC}(t)]$ is the vector of the instantaneous value of all capacitor voltages and all inductor currents

- Transient simulations are decomposed into a sequence of individual problems where the output of all energy storage elements at step k serves as the initial condition (not initial guess) of all energy storage elements at step $k+1$
- Simulator attempts to conserve charge in each simulation step. This is a key concept that must be incorporated for consistency in any circuit simulator when L s and C s are included in schematic.

Quirks with Circuit Simulators

Transient Simulation Approach in Simulators



Simulator attempts to conserve charge in each simulation step (KCL and KVL)

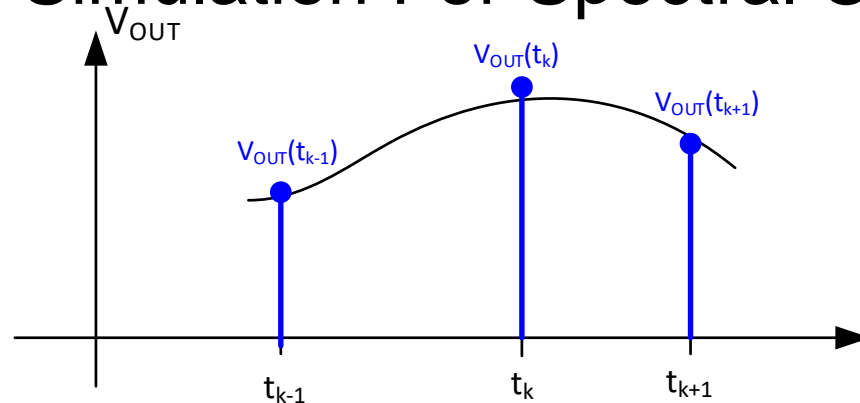
Basic physical principle that is integral to any transient simulation of circuits with energy storage elements: **CHARGE IS CONSERVED !!**

Is charge conserved in a circuit simulator when doing transient simulations?

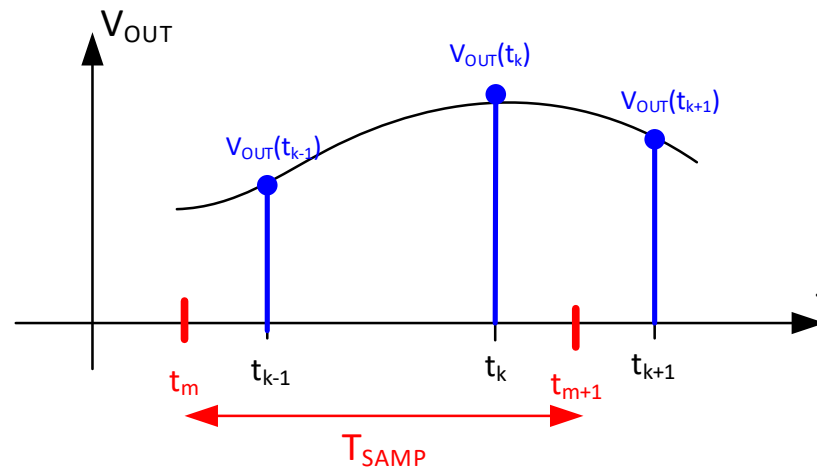
No ! It is only conserved locally (in individual time intervals) so major divergence can occur in some extreme situations due to accumulative round off effects

Quirks with Circuit Simulators

Transient Simulation For Spectral Characterization



Normal time-stepping algorithm is used to obtain transient response

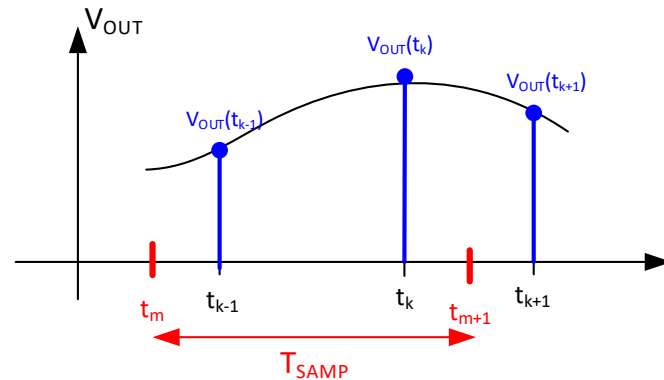


DFT requires output at precisely uniformly spaced predetermined points, t_m, t_{m+1}, \dots

These points are almost never coincident with the time-stepping points !

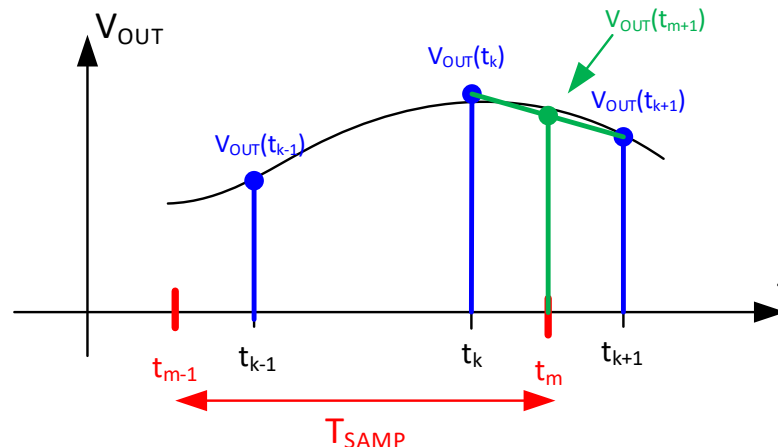
Quirks with Circuit Simulators

Transient Simulation For Spectral Characterization



So how does the simulator generate outputs at the required time points ?

It interpolates ! Exact interpolation algorithm may vary from vendor to vendor

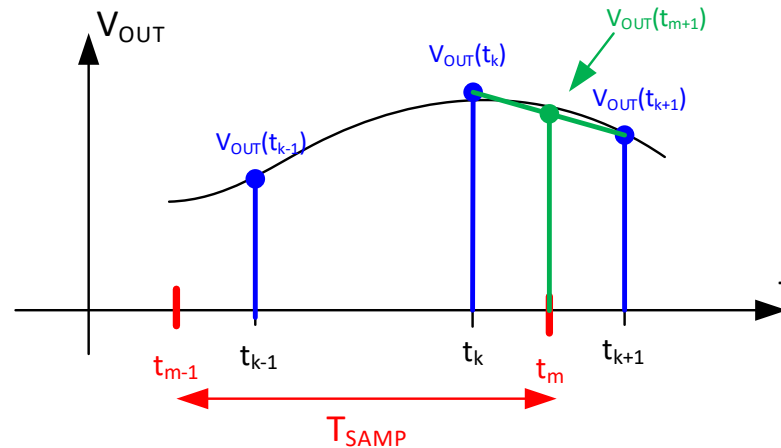


For linear interpolation

$$V_{OUT}(t_m) = V_{OUT}(t_k) + [V_{OUT}(t_{k+1}) - V_{OUT}(t_k)] \frac{t_{m+1} - t_k}{t_{k+1} - t_k}$$

Quirks with Circuit Simulators

Transient Simulation For Spectral Characterization



For linear interpolation

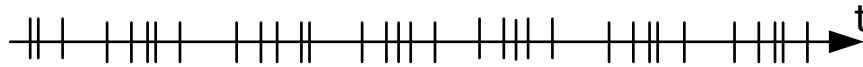
$$V_{OUT}(t_m) = V_{OUT}(t_k) + [V_{OUT}(t_{k+1}) - V_{OUT}(t_k)] \frac{t_{m+1} - t_k}{t_{k+1} - t_k}$$

What errors are introduced in determining $V_{OUT}(t_m)$?

- Errors in calculating $V_{OUT}(t_k)$
 - (most problematic for long simulations with multiple energy storage elements)
 - (may not be particularly problematic for spectral characterization since N_p often not too long)
 - (must simulate long enough for natural response to die out if ALL initial conditions are not correctly set)
- Errors associated with interpolation $V_{OUT}(t_k)$

Quirks with Circuit Simulators

If interpolation is a problem, what can be done about it



For linear interpolation

$$V_{OUT}(t_m) = V_{OUT}(t_k) + [V_{OUT}(t_{k+1}) - V_{OUT}(t_k)] \frac{t_{m+1} - t_k}{t_{k+1} - t_k}$$

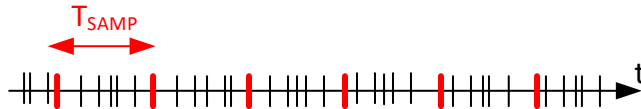
- specify the sample points where you want output?



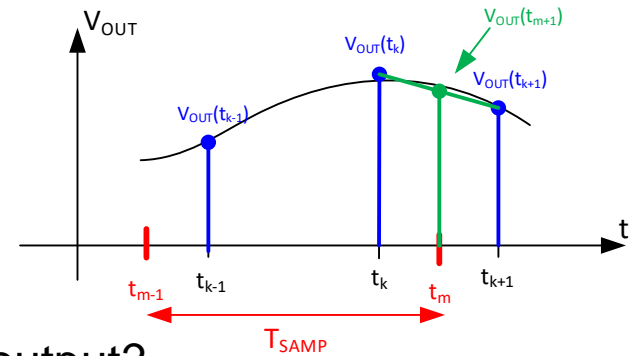
- specify a very small maximum step size



- modify time stepping algorithm to include desired sample points



- Add noninteracting waveform with steep slope at desired transition points



No – it will ignore such request

May help but maybe not enough, long sim times, possibly convergence problems

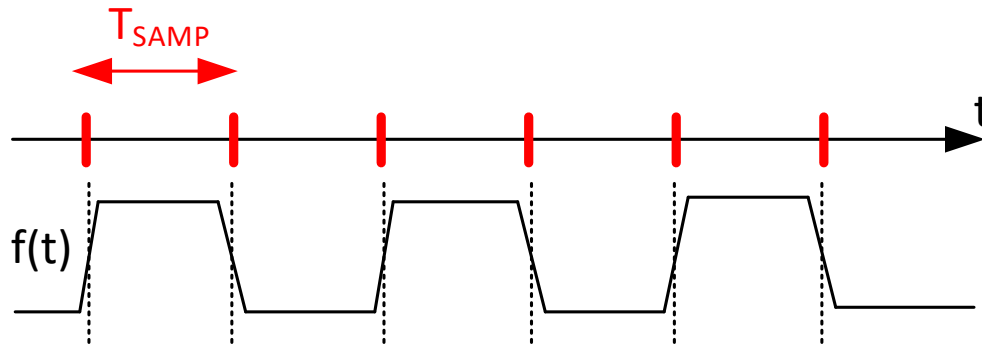
May help a lot ! Use Strobe Period function if available in simulator

May also help a lot ! Forces some time stepping algorithms to take sample near sample points

Quirks with Circuit Simulators

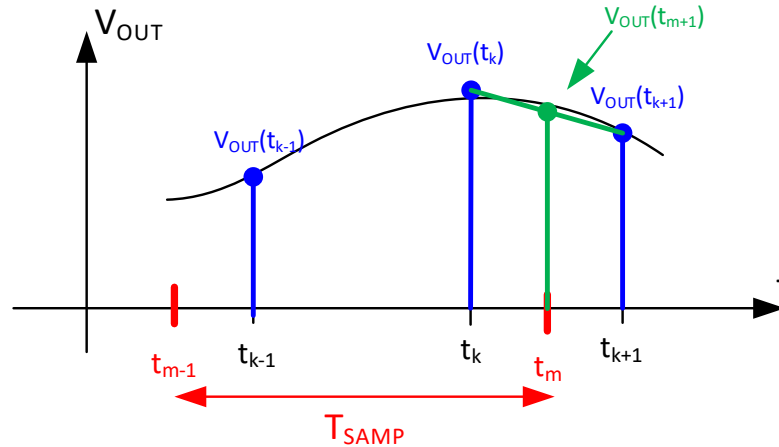
If interpolation is a problem, what can be done about it

- Add noninteracting waveform with steep slope at desired transition points



Quirks with Circuit Simulators

Transient Simulation For Spectral Characterization



Errors associated with interpolation $V_{OUT}(t_k)$

Will consider example using Spectre

Do not know what the interpolation algorithm is

Will not include any energy storage elements

Spectre Limitations in Spectral Analysis

Thanks to Xilu Wang for simulation results

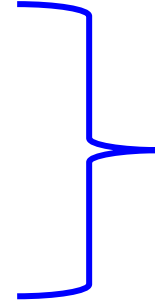
- Normal Transient Analysis
- Strobe Period Timing
- Coherent Sampling

Simulation Conditions

$$V(t) = \sin(2\pi \cdot 50t)$$

11 periods

Coherent Sampling



Number of Samples:

- 512
- 4096

Type of Samples:

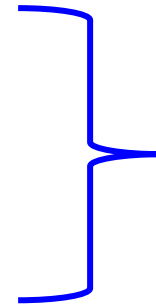
- Standard Sweep
- Strobe Period Sweep

512 Samples with Standard Sweep

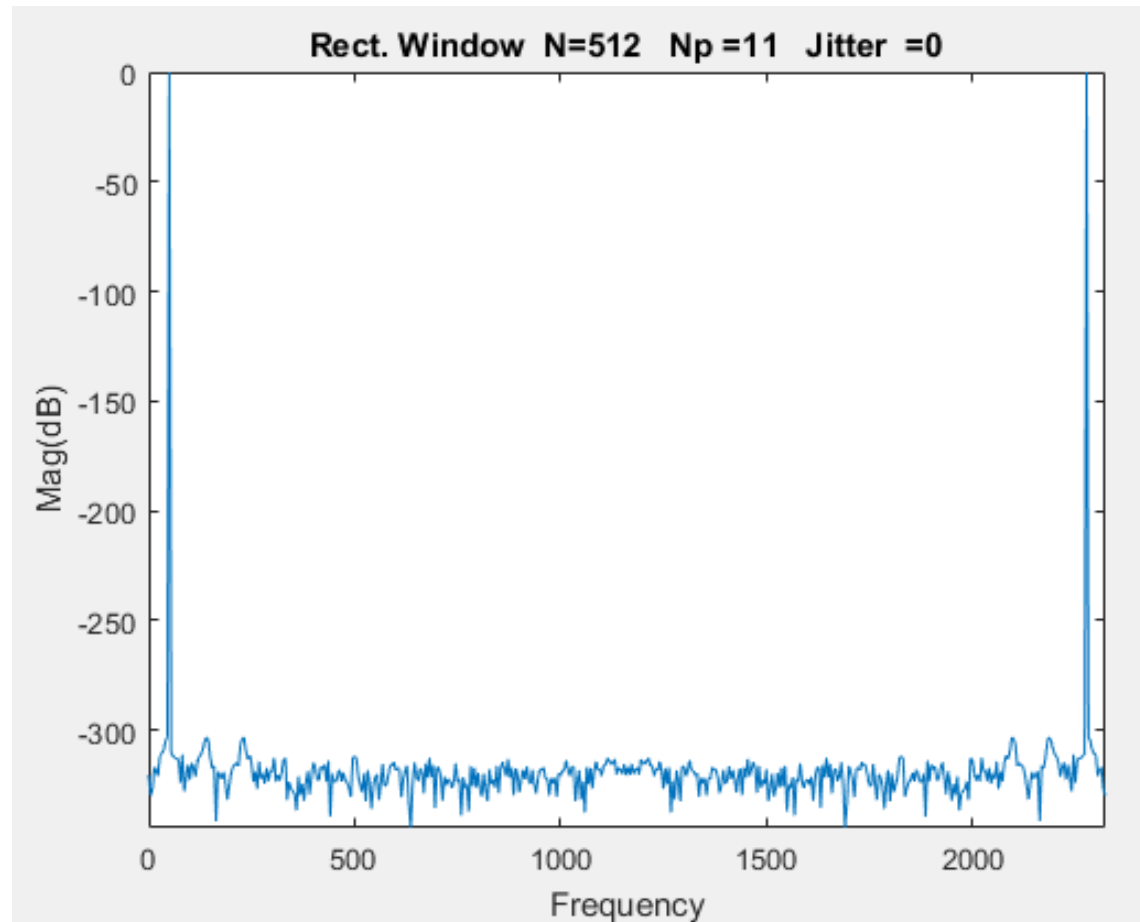
$$V(t) = \sin(2\pi \cdot 50t)$$

11 periods

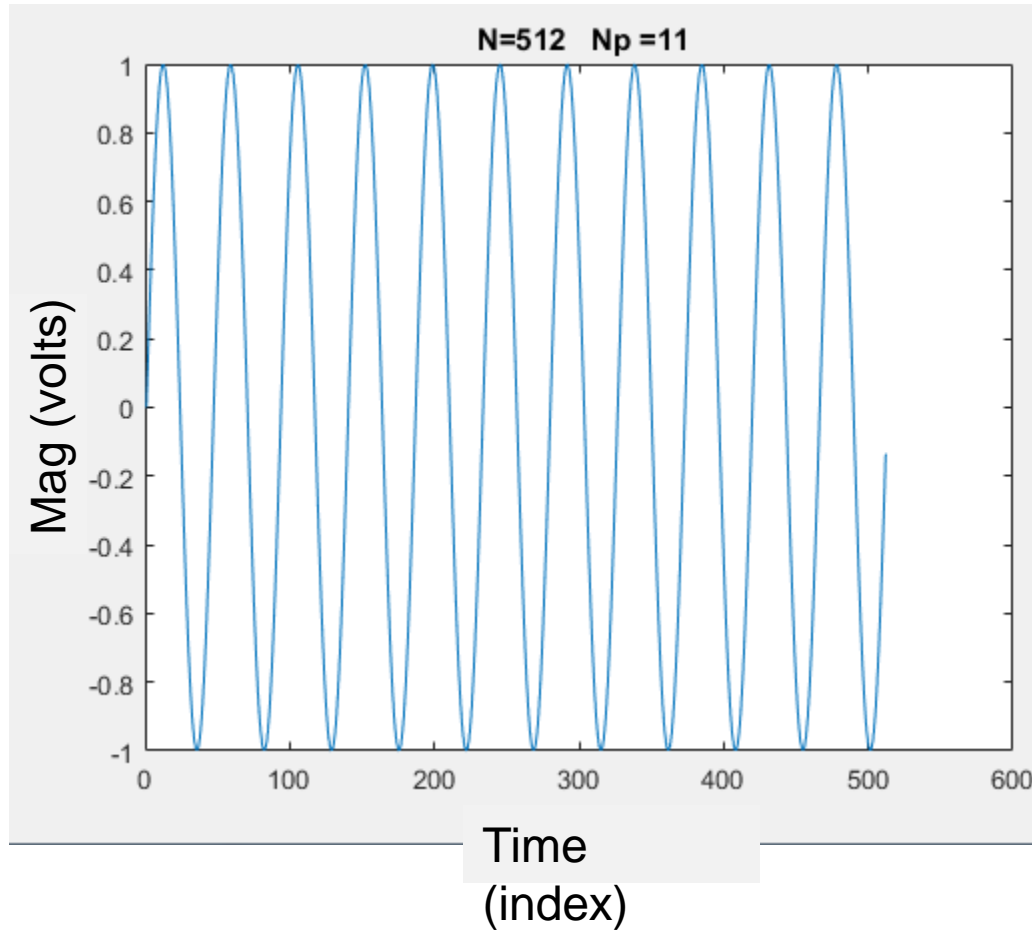
Coherent Sampling



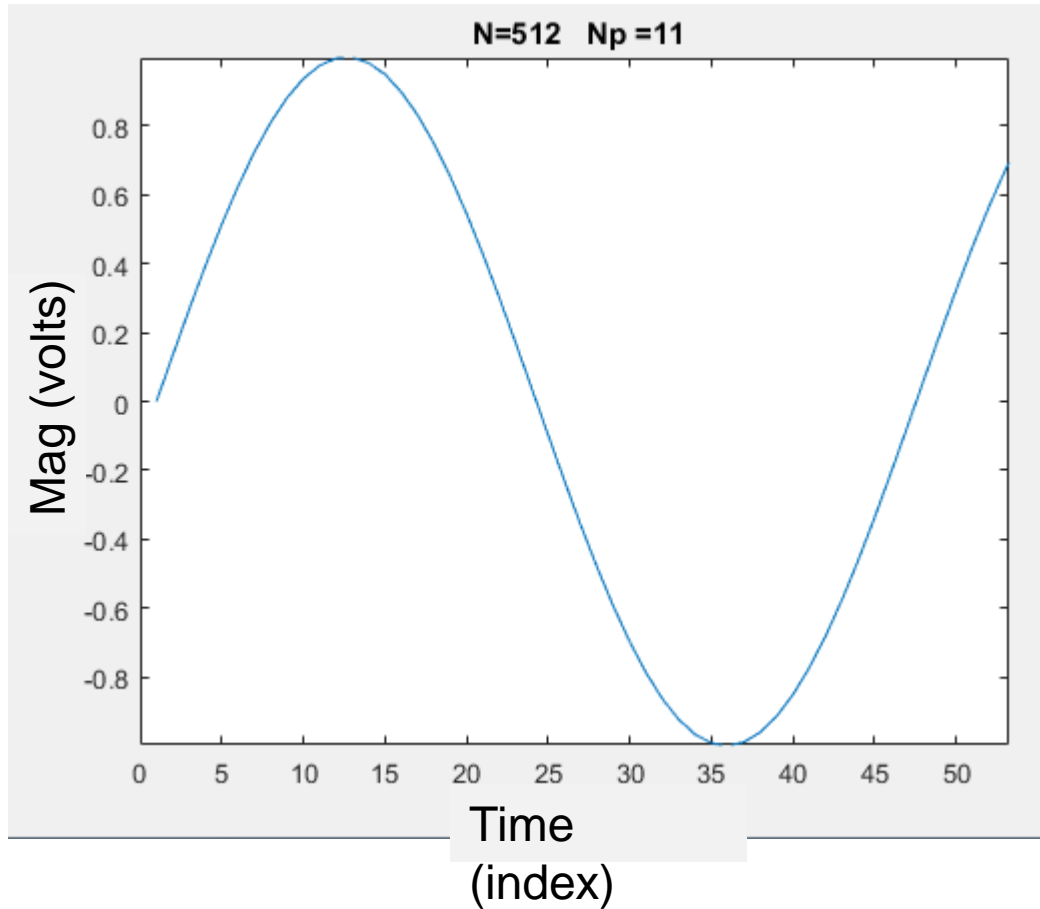
For reference: Results obtained with MatLab for $N=512$



512 Samples with Standard Sweep



512 Samples with Standard Sweep



512 Samples with Standard Sweep

```
Pyyt1 =
```

```
Columns 1 through 12
```

```
-107.8981 -94.1331 -108.0317 -98.4157 -108.4194 -94.7571 -108.9965 -109.0037 -109.6020 -105.0095 -110.0367 -0.0025
```

```
Columns 13 through 24
```

```
-110.1847 -112.5300 -110.0274 -124.3662 -109.6023 -95.7592 -109.0660 -108.3765 -108.6183 -96.3597 -108.3281 -101.9470
```

```
Columns 25 through 36
```

```
-108.2245 -127.5677 -108.3644 -92.2404 -108.7400 -115.5801 -109.2418 -106.8007 -109.7118 -79.7378 -109.9937 -111.7277
```

```
Columns 37 through 48
```

```
-109.9798 -111.2019 -109.6609 -95.7148 -109.1668 -108.6139 -108.7231 -95.8926 -108.4382 -91.2831 -108.3050 -110.9404
```

512 Samples with Standard Sweep

Columns 49 through 60

-108.3530 -92.7636 -108.6349 -115.7449 -109.1516 -112.1106 -109.8730 -84.9671 -110.7375 -108.1262 -111.5945 -104.6449

Columns 61 through 72

-112.1962 -95.5508 -112.3197 -108.1874 -112.0368 -91.3543 -111.6457 -95.5037 -111.3708 -107.0414 -111.3212 -93.7515

Columns 73 through 84

-111.5009 -110.5545 -111.8098 -101.0709 -112.1458 -89.9120 -112.5548 -112.9274 -113.1741 -117.4284 -114.0466 -97.7820

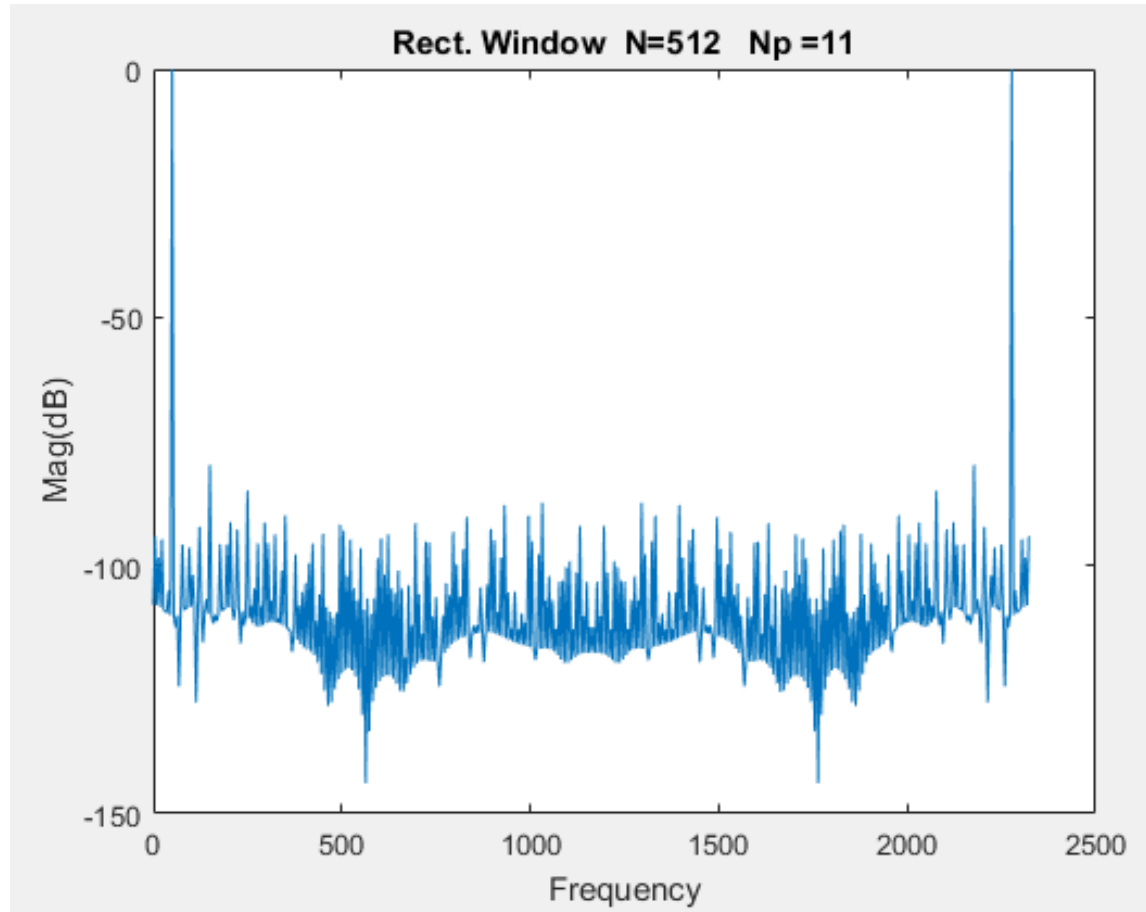
Columns 85 through 96

-115.0135 -108.1029 -115.8627 -105.3494 -116.4956 -103.5153 -116.9316 -99.5311 -117.3540 -95.6142 -118.0994 -111.2058

Columns 97 through 108

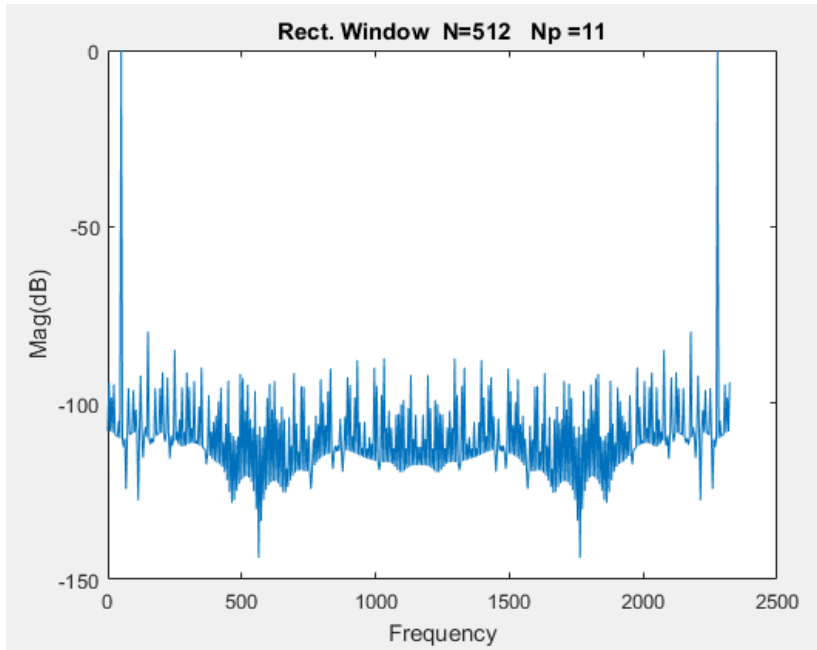
-119.5251 -105.8620 -121.8988 -93.6210 -125.2388 -108.3341 -128.2779 -109.3118 -127.5511 -110.5460 -124.9168 -109.4982

512 Samples with Standard Sweep

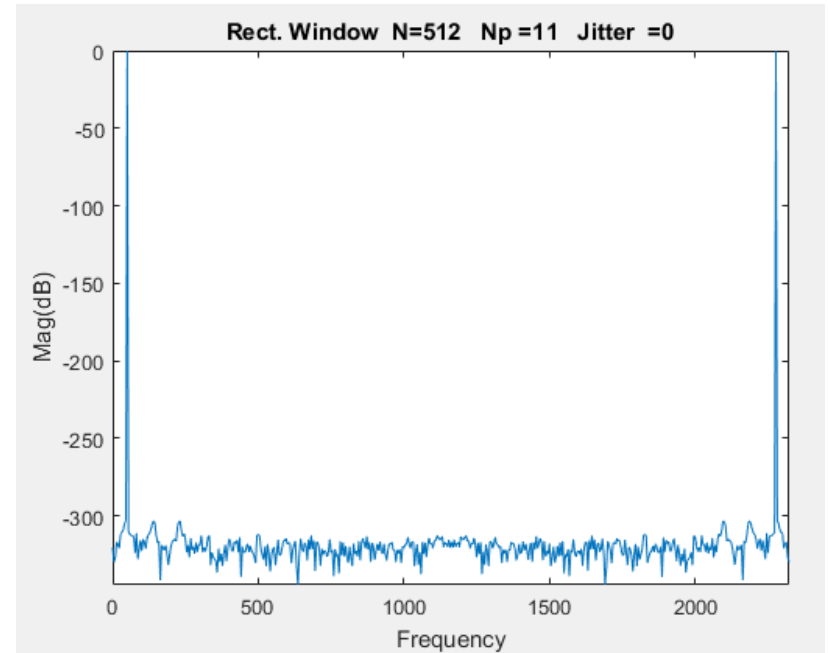


- Note dramatic increase in noise floor
- Note what appear to be some harmonic terms extending above noise floor

MatLab comparison: 512 Samples with Standard Sweep

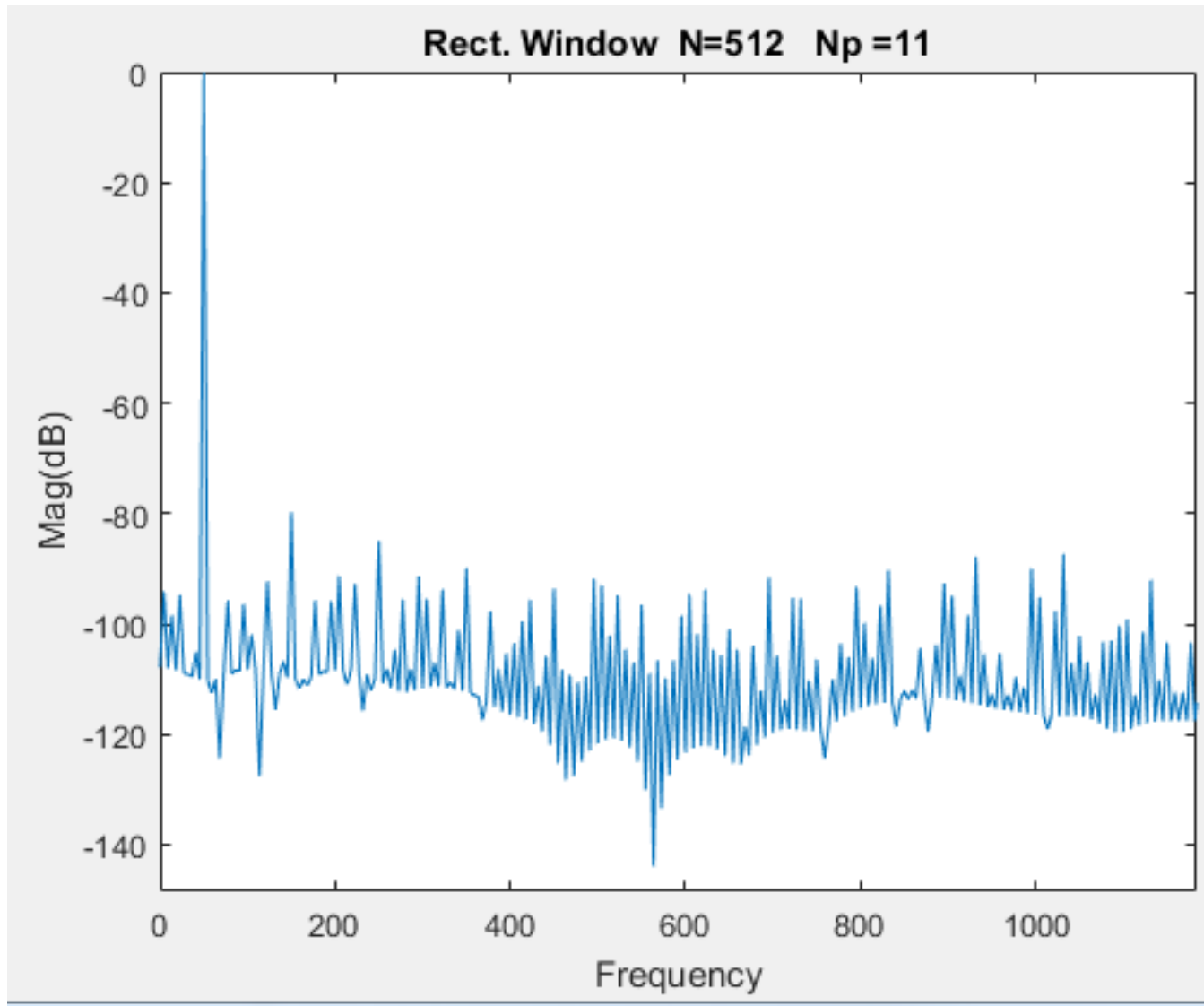


Spectre Results

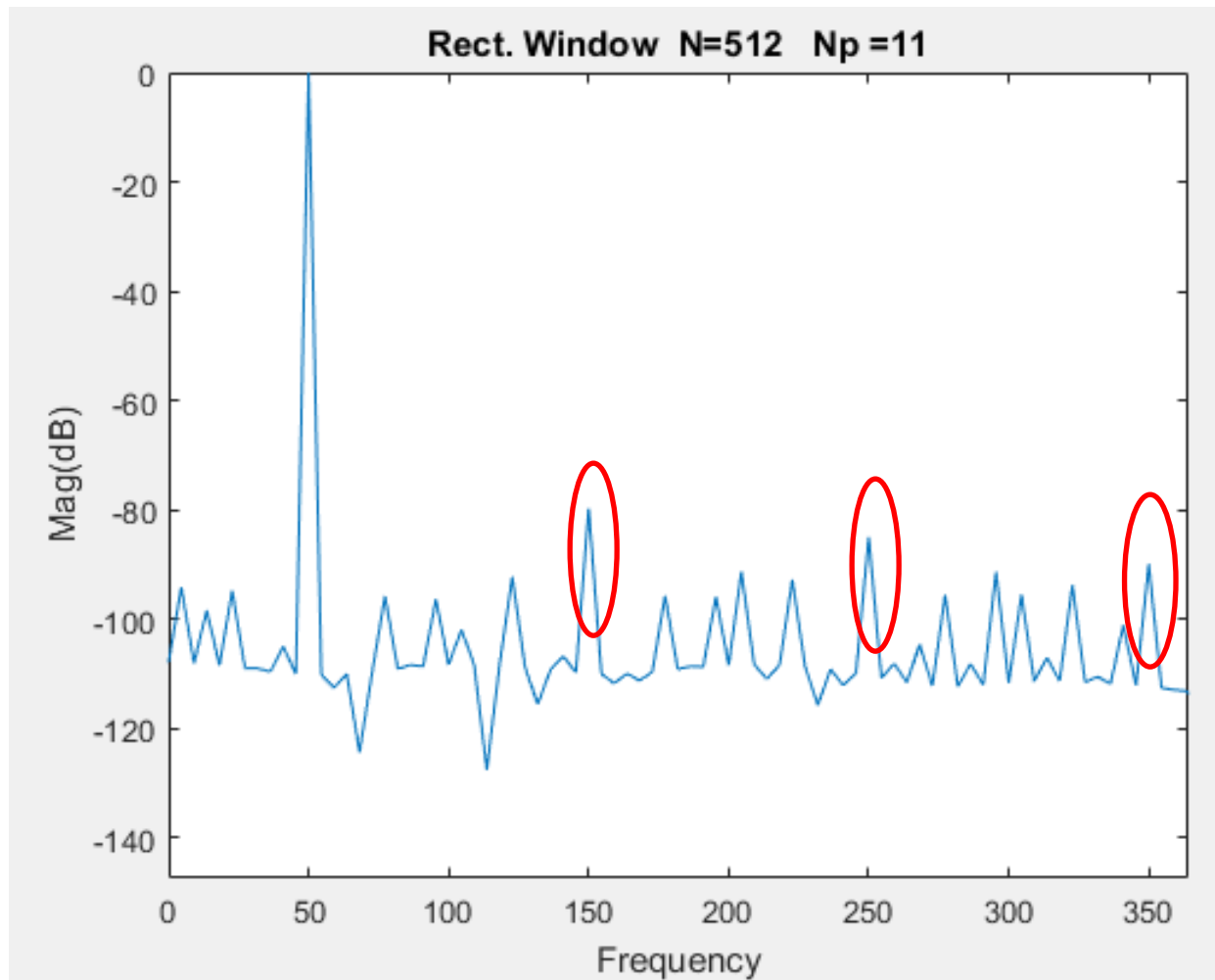


MatLab Results

512 Samples with Standard Sweep

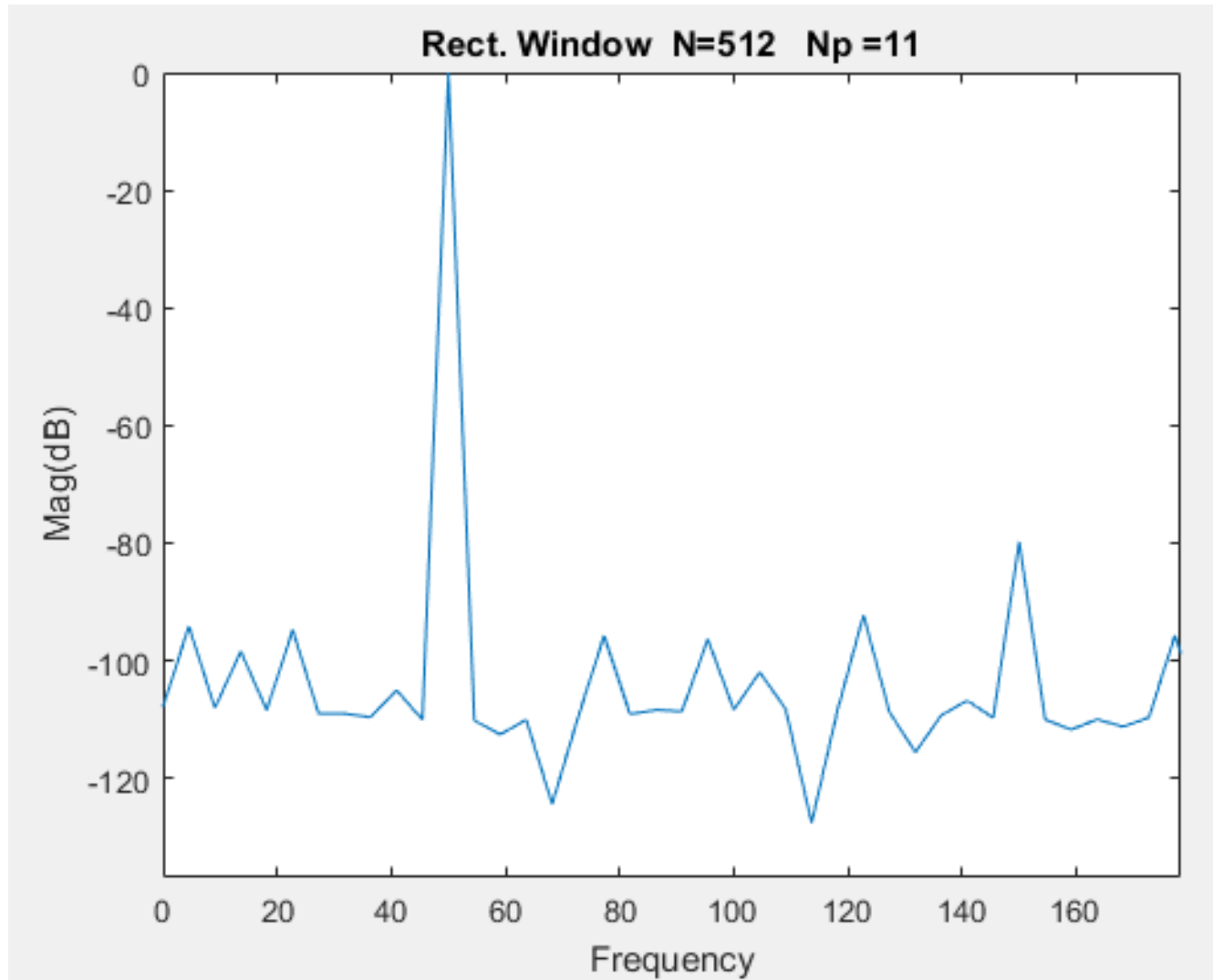


512 Samples with Standard Sweep



Note presence of odd harmonics in spectrum

512 Samples with Standard Sweep

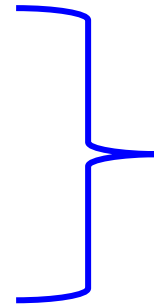


512 Samples with Strobe Period Sweep

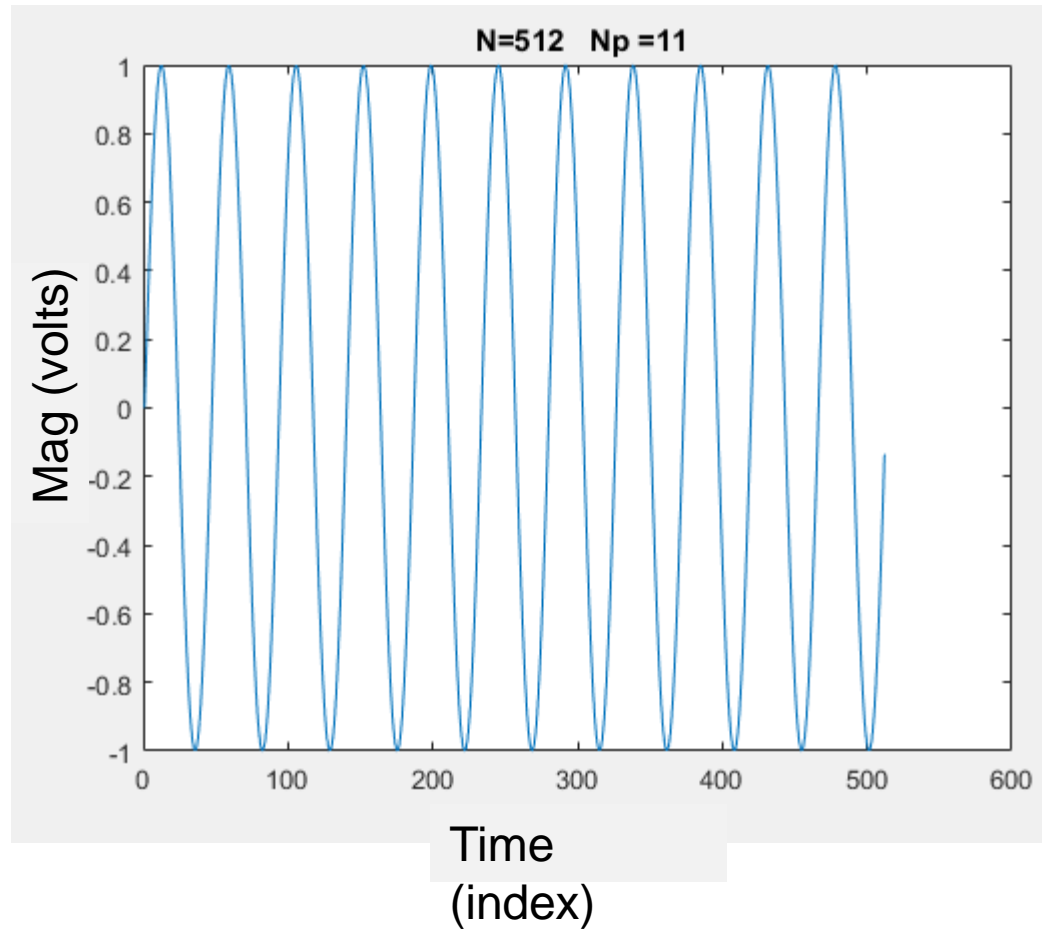
$$V(t) = \sin(2\pi \cdot 50t)$$

11 periods

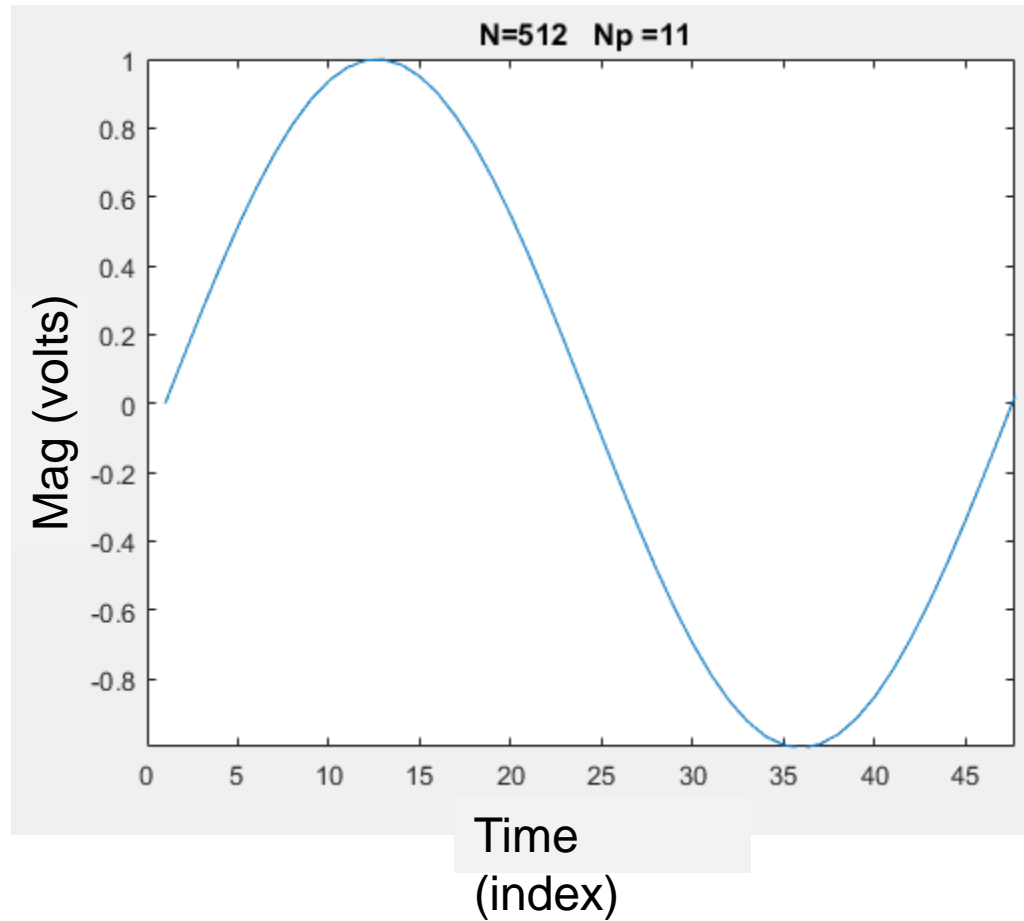
Coherent Sampling



512 Samples with Strobe Period



512 Samples with Strobe Period



512 Samples with Strobe Period

Ppyt2 =

Columns 1 through 12

-289.9823 -277.1621 -271.8971 -269.7867 -287.1463 -274.9517 -274.1616 -268.2808 -286.3367 -270.6890 -264.2517 0.0000

Columns 13 through 24

-264.9125 -263.7218 -269.4627 -281.3076 -273.7005 -264.8333 -273.1021 -274.2804 -275.7987 -282.2465 -273.4191 -272.4758

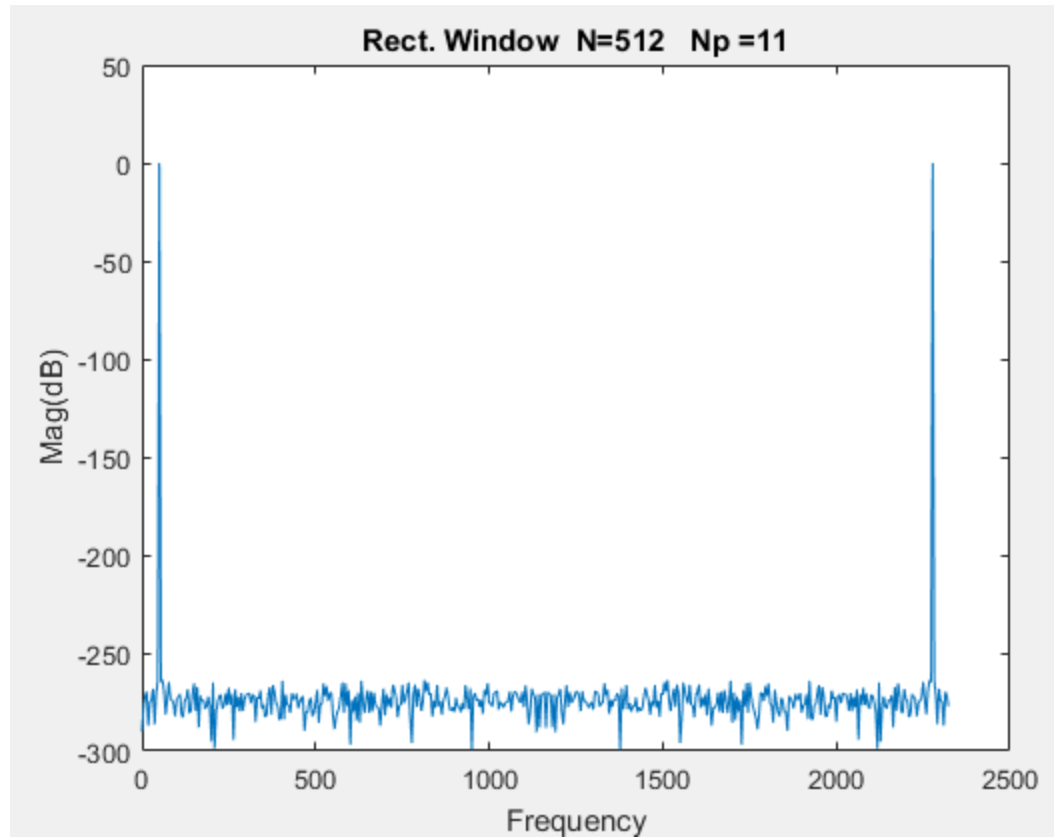
Columns 25 through 36

-270.9356 -281.3558 -282.8219 -274.2676 -273.4832 -268.4204 -280.8477 -279.3164 -270.8593 -265.4445 -279.4813 -267.3563

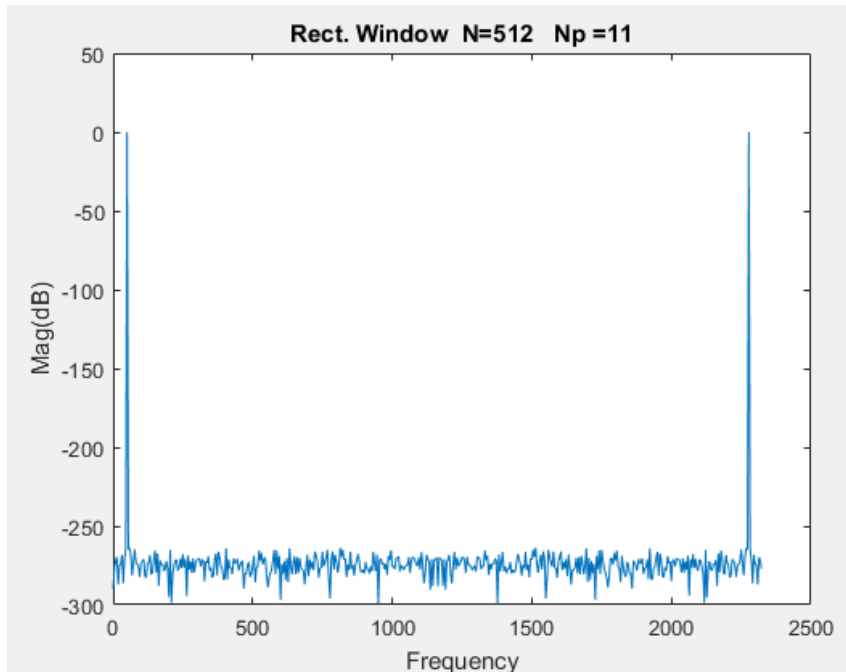
Columns 37 through 48

-288.0473 -271.6637 -274.6875 -274.0074 -278.3424 -277.6395 -272.2091 -276.9768 -295.2068 -265.0774 -298.7341 -280.0345

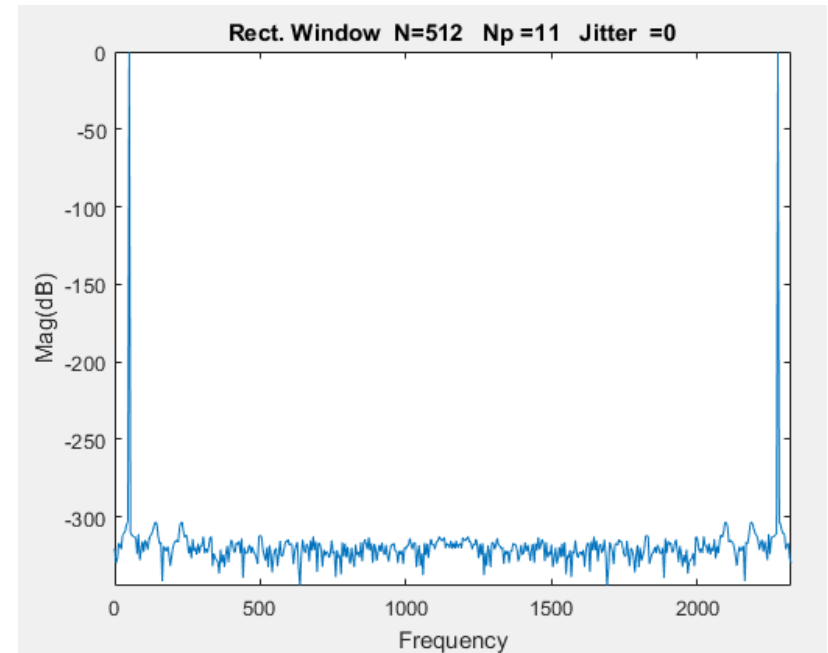
512 Samples with Strobe Period



MatLab comparison: 512 Samples with Strobe Period Sweep

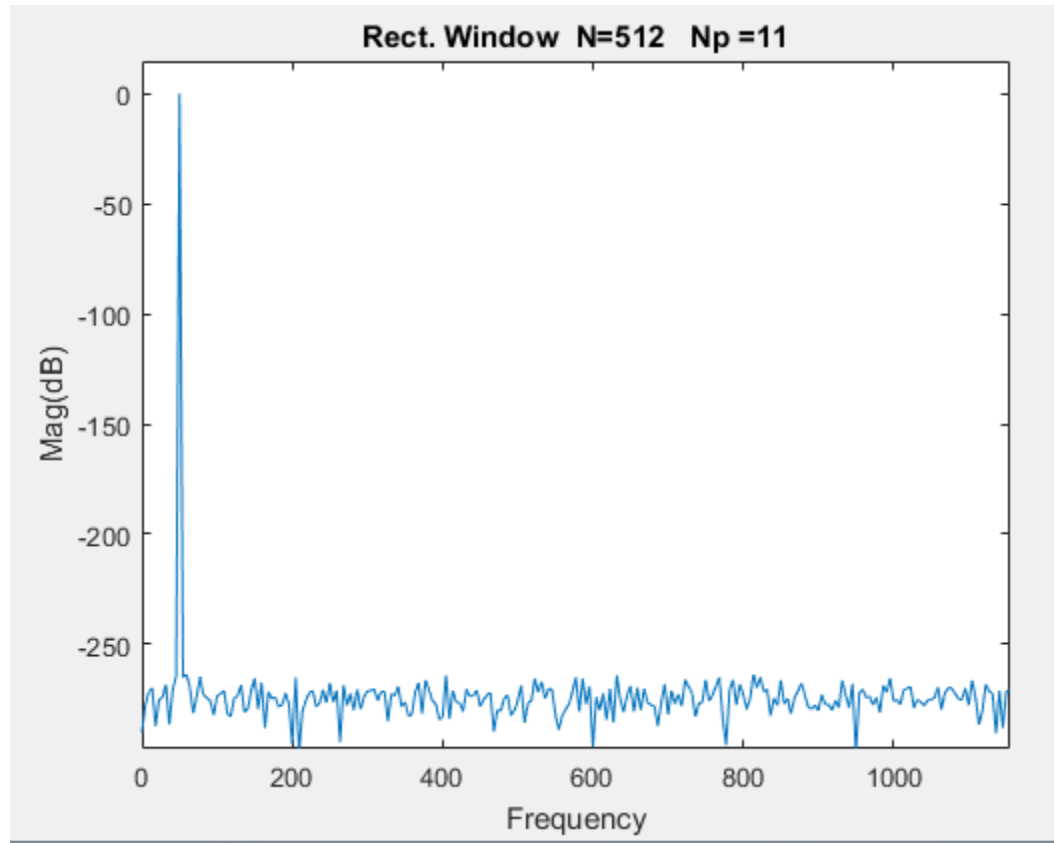


Spectre Results

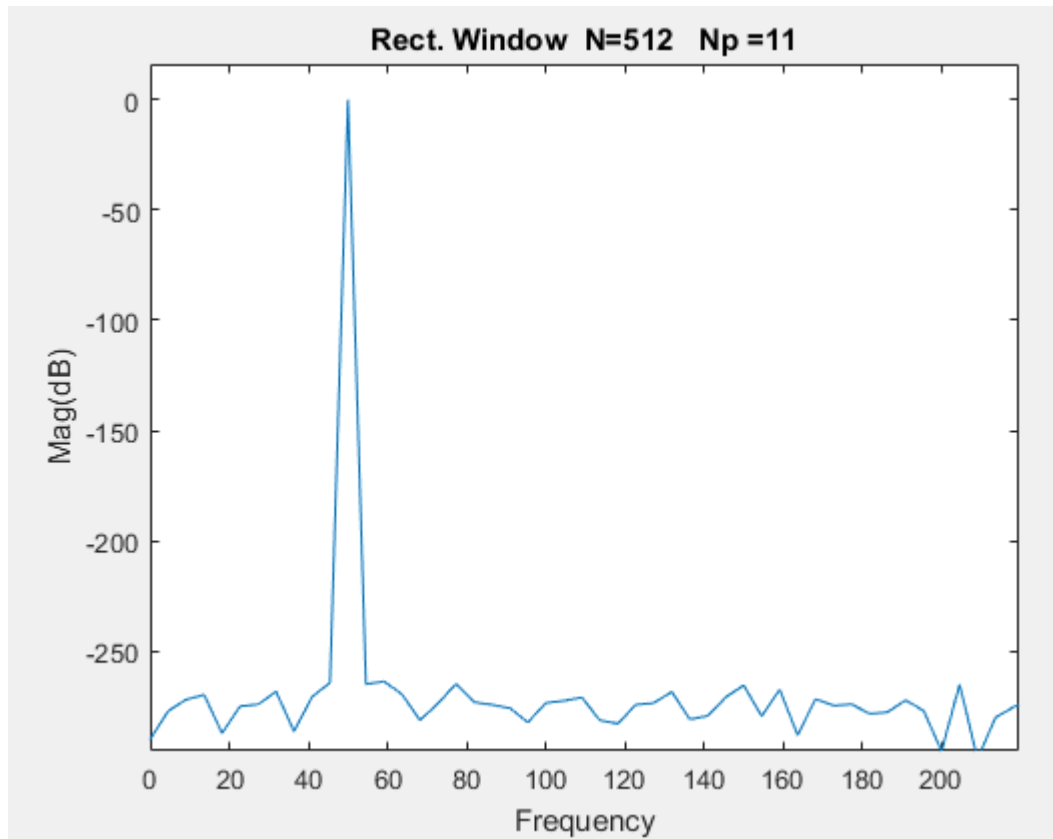


MatLab Results

512 Samples with Strobe Period



512 Samples with Strobe Period

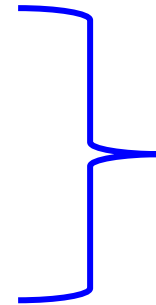


4096 Samples with Standard Sweep

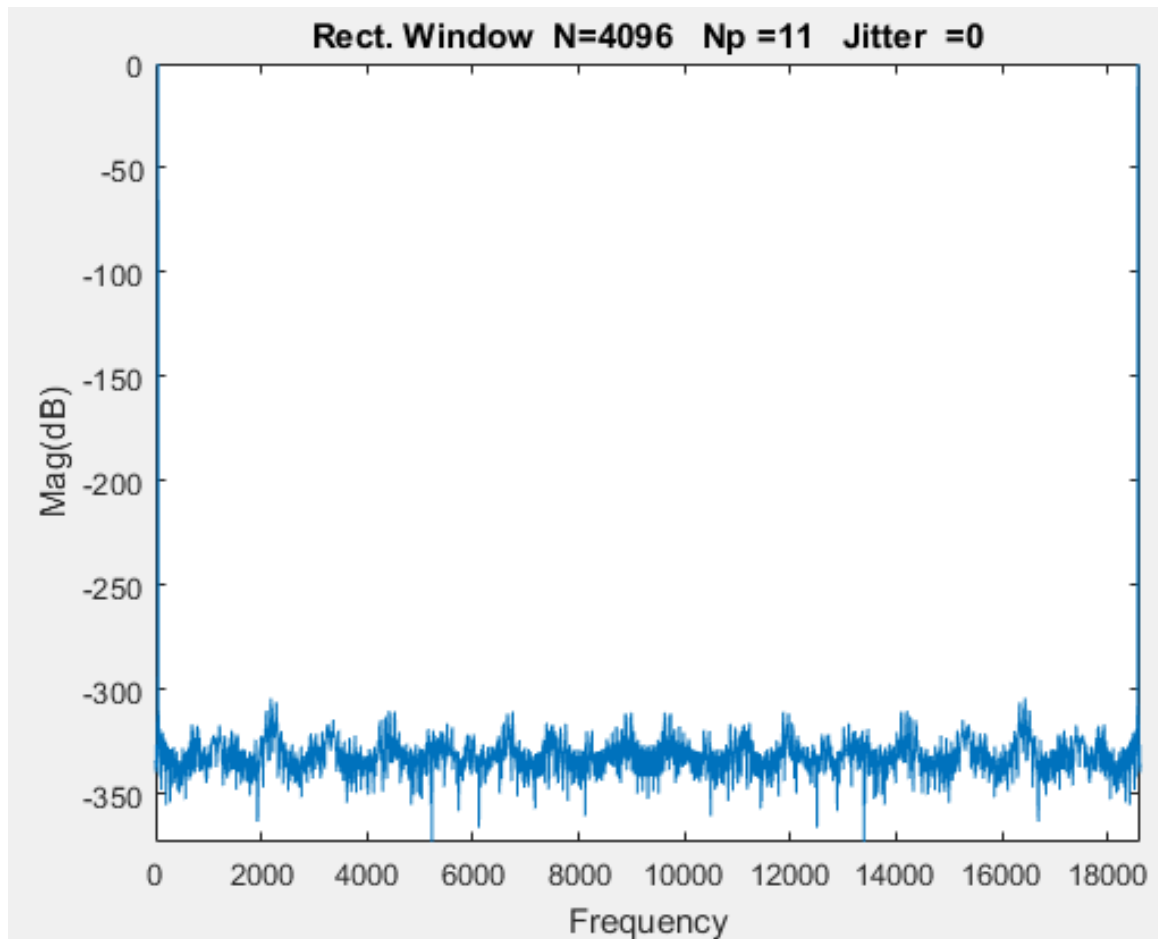
$$V(t) = \sin(2\pi \cdot 50t)$$

11 periods

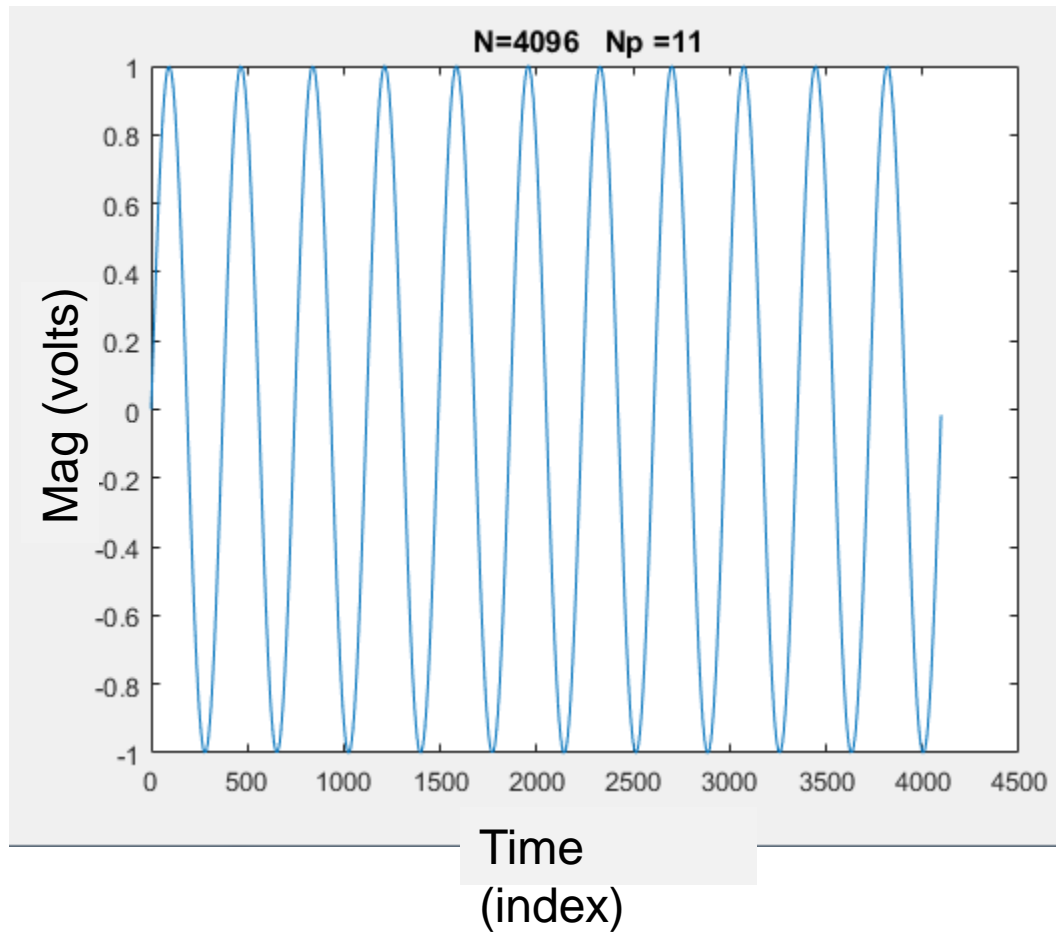
Coherent Sampling



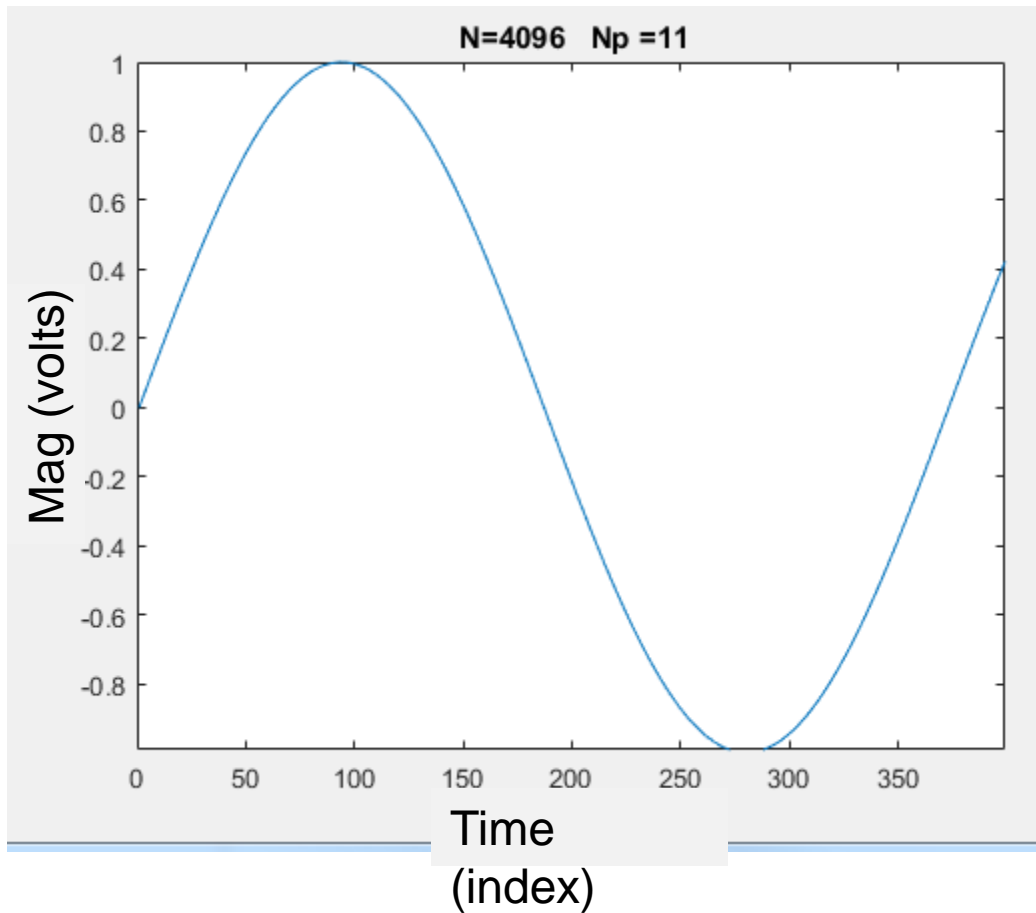
For reference: Results obtained with MatLab for $N=4096$



4096 Samples with Standard Sweep



4096 Samples with Standard Sweep



4096 Samples with Standard Sweep

```
Pyyt3 =
```

```
Columns 1 through 12
```

```
-108.3243 -108.1538 -108.2707 -108.0618 -108.1535 -108.0412 -108.0422 -109.2666 -107.9762 -107.8815 -107.9734 -0.0024
```

```
Columns 13 through 24
```

```
-108.0437 -108.0723 -108.1927 -109.4684 -108.4265 -108.3864 -108.7503 -107.7815 -109.1369 -109.1277 -109.4911 -109.7293
```

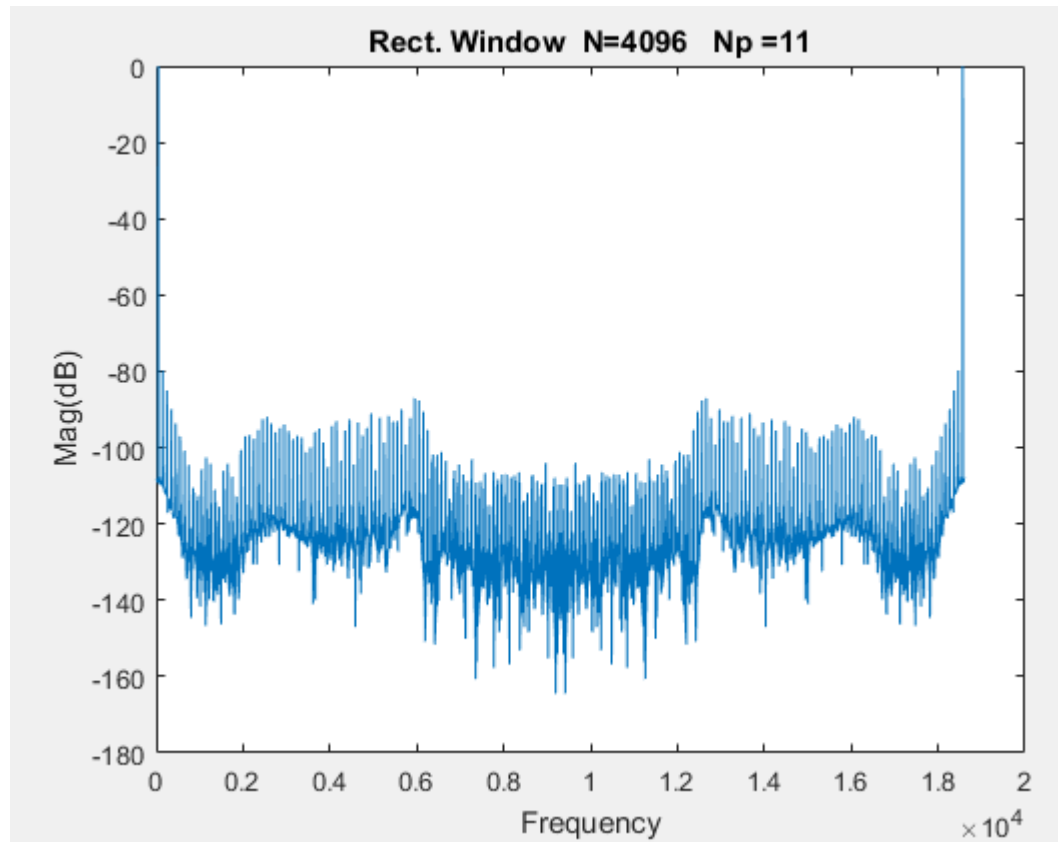
```
Columns 25 through 36
```

```
-109.7098 -108.8871 -109.7944 -109.4286 -109.8171 -108.5378 -109.8456 -109.8612 -109.9177 -79.7470 -110.0501 -110.1301
```

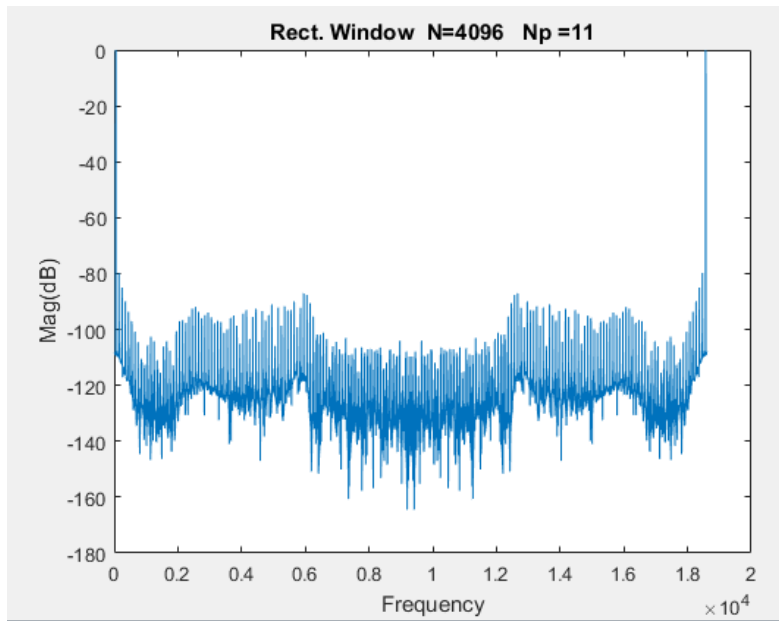
```
Columns 37 through 48
```

```
-110.2518 -104.6517 -110.5298 -110.8421 -110.8907 -111.4034 -111.3159 -111.0945 -111.7096 -111.7185 -111.9500 -111.0991
```

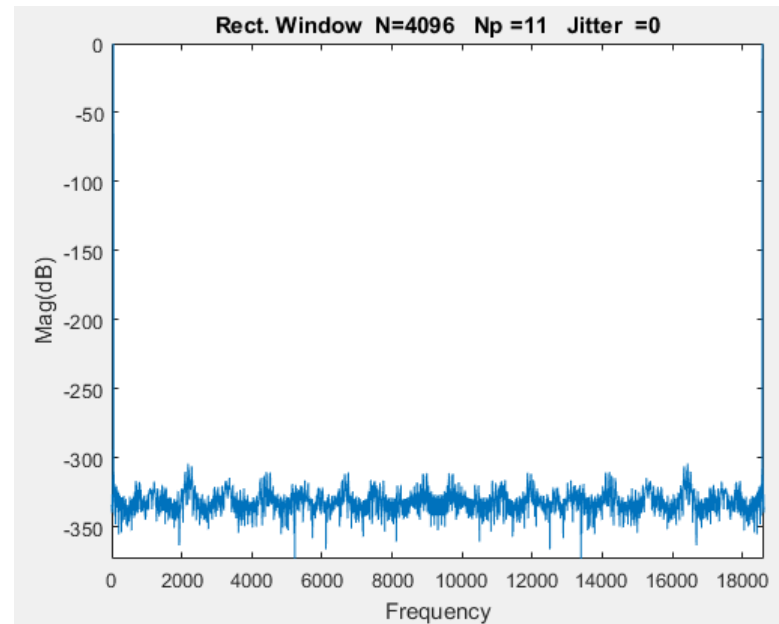
4096 Samples with Standard Sweep



Comparison 4096 Samples with Standard Sweep

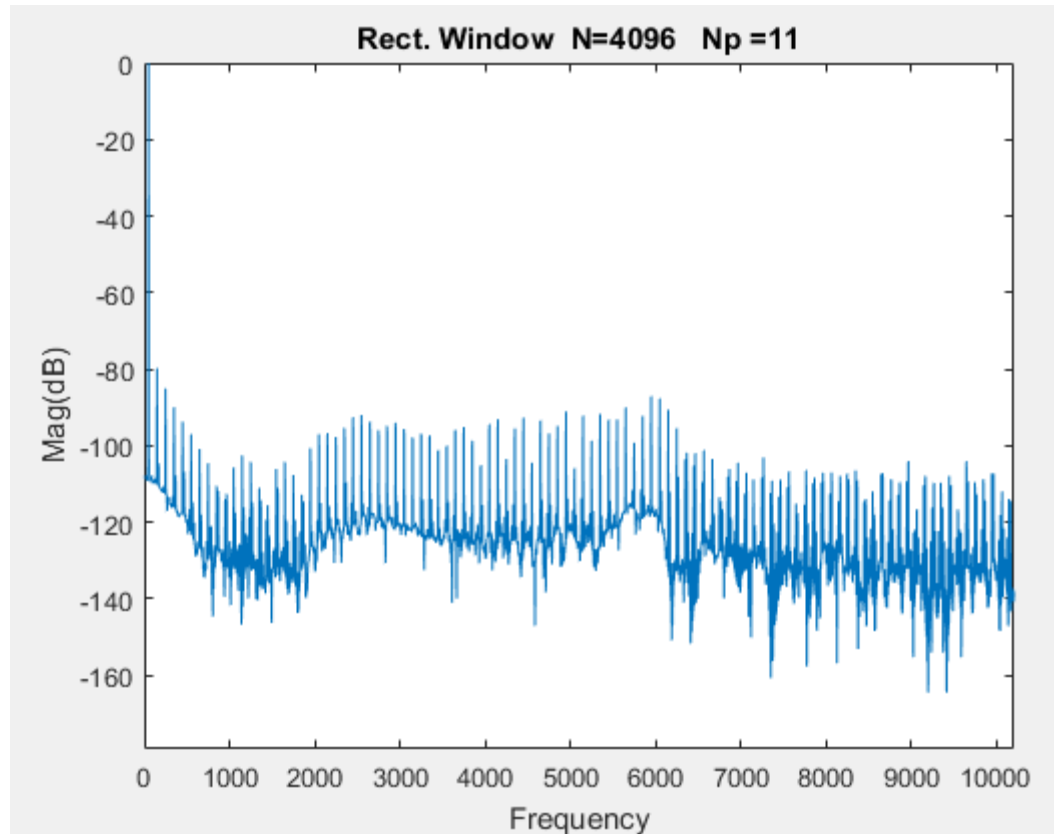


Spectre

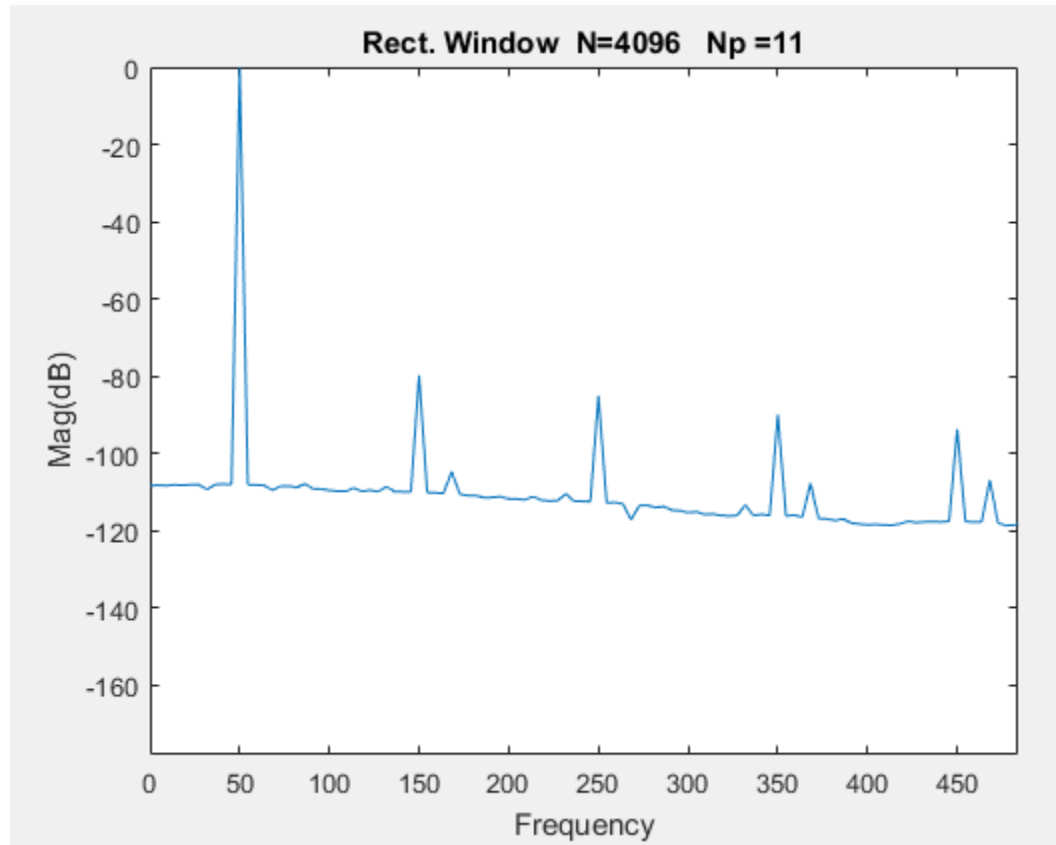


MatLab

4096 Samples with Standard Sweep

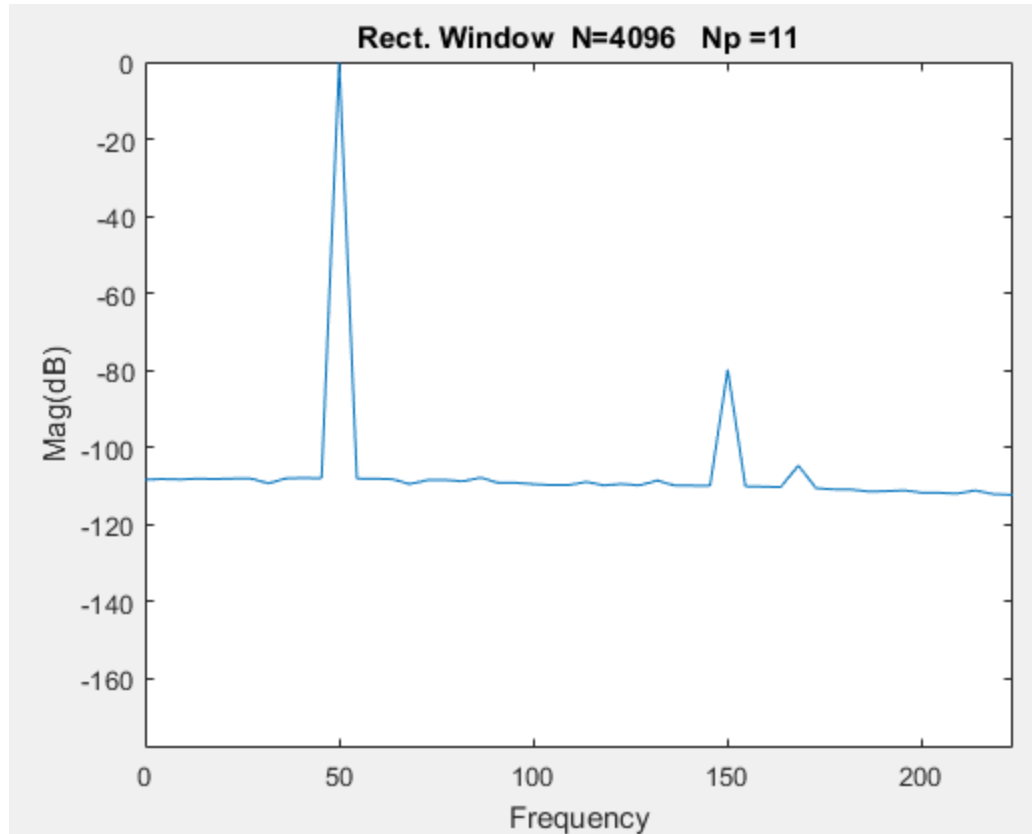


4096 Samples with Standard Sweep



Note presence of odd harmonics in spectrum

4096 Samples with Standard Sweep

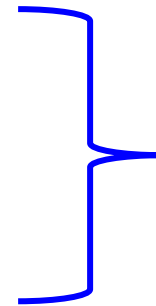


4096 Samples with Strobe Period Sweep

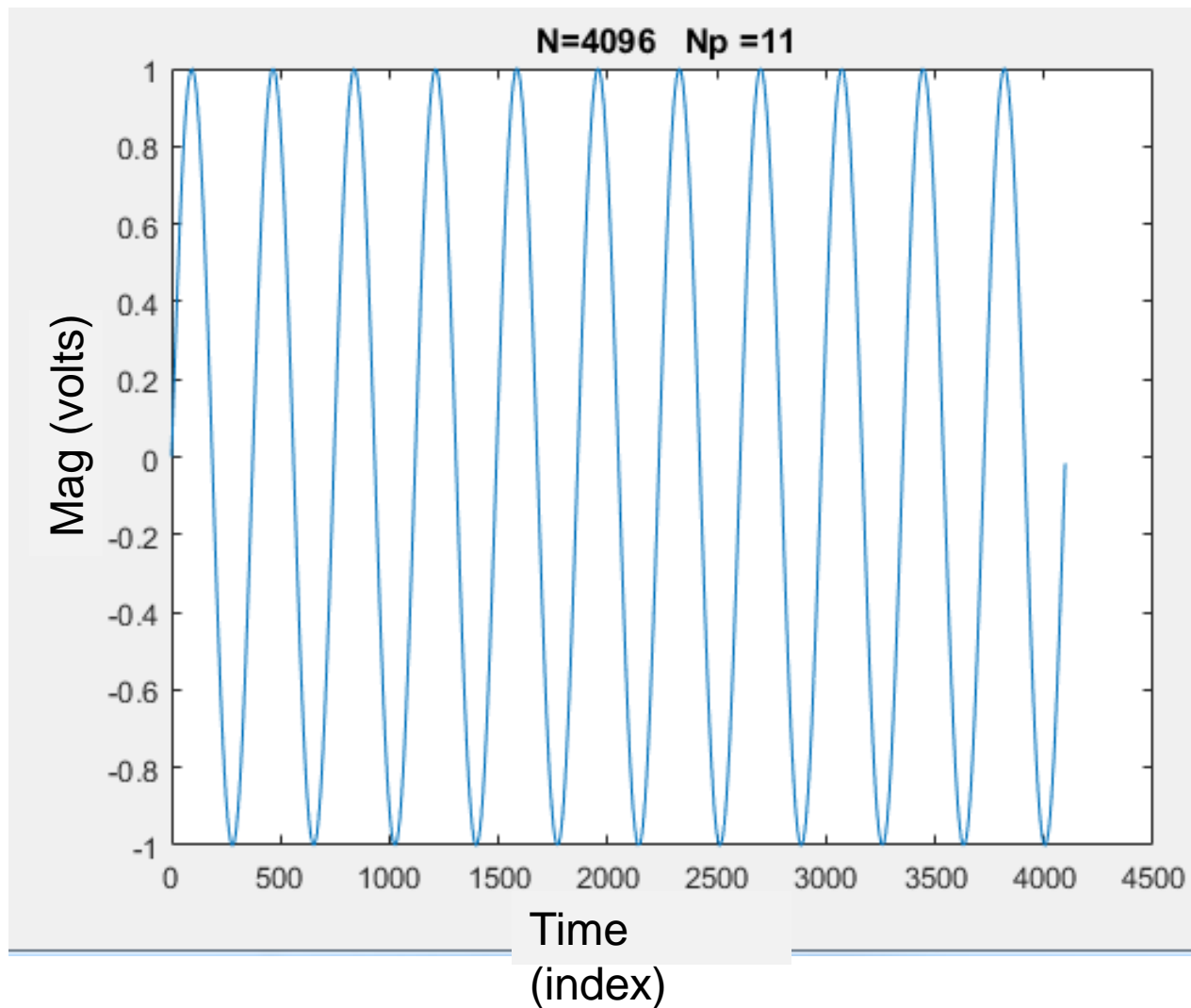
$$V(t) = \sin(2\pi \cdot 50t)$$

11 periods

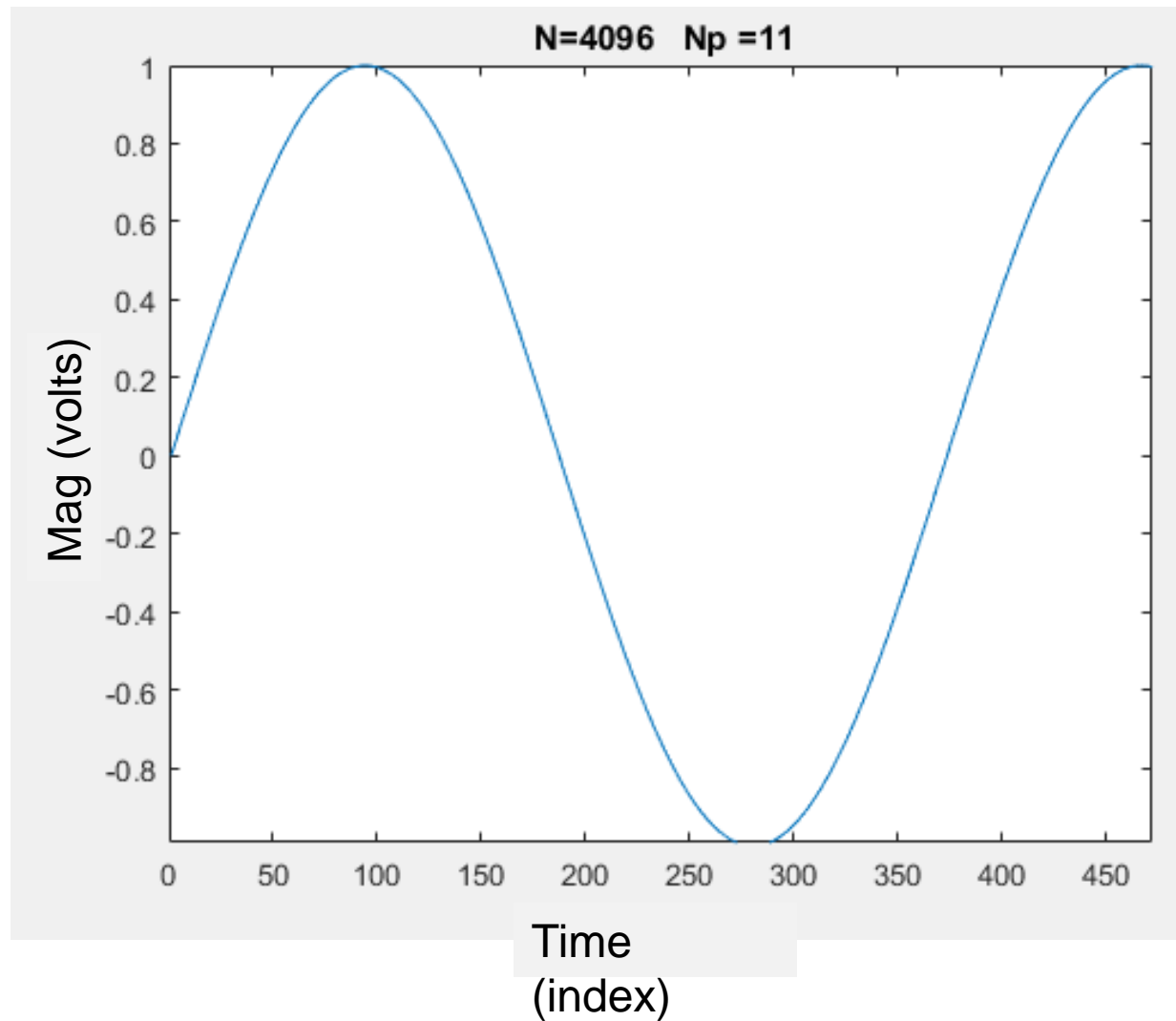
Coherent Sampling



4096 Samples with Strobe Period



4096 Samples with Strobe Period



4096 Samples with Strobe Period

```
Pyyt4 =
```

```
Columns 1 through 12
```

```
-275.7993 -274.2288 -271.2122 -266.4879 -261.9356 -260.8993 -256.6778 -254.4684 -252.1418 -247.4997 -238.6334 -0.0000
```

```
Columns 13 through 24
```

```
-237.9355 -245.9934 -249.3333 -250.9084 -252.6585 -254.7659 -255.1826 -256.4114 -256.4550 -258.0311 -258.5182 -259.4258
```

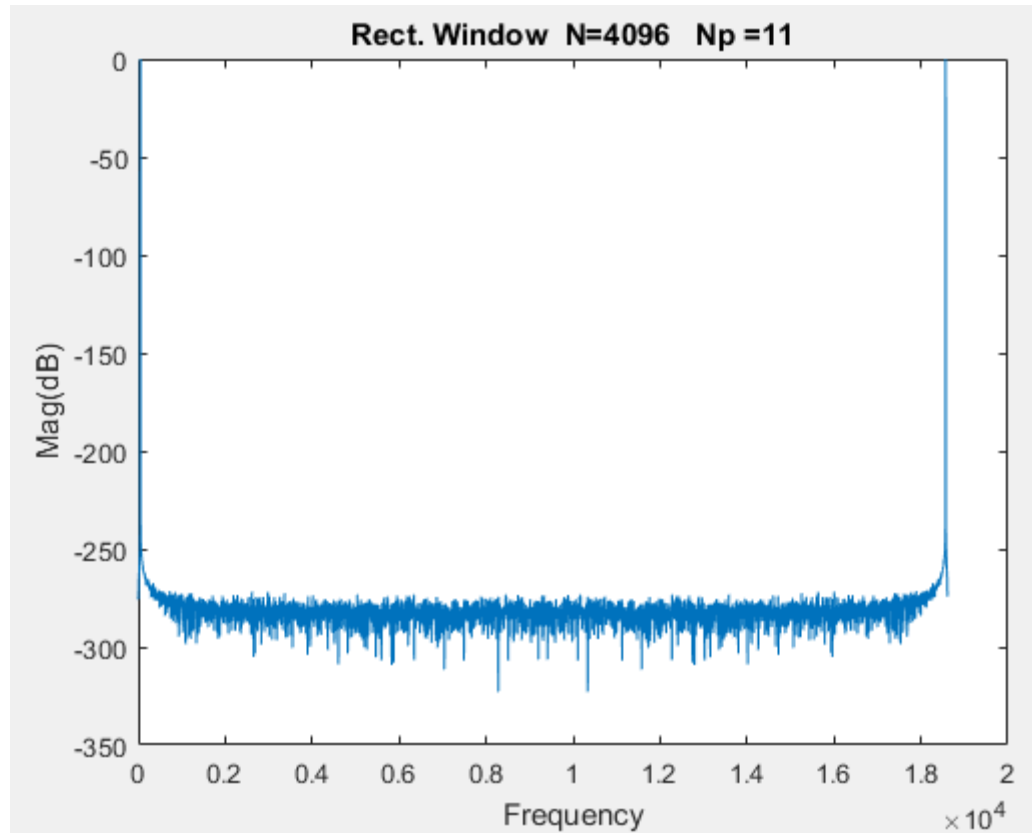
```
Columns 25 through 36
```

```
-260.1189 -260.6618 -260.7491 -261.8162 -261.6129 -262.3357 -262.2738 -263.3625 -264.1235 -263.3313 -262.6442 -264.9796
```

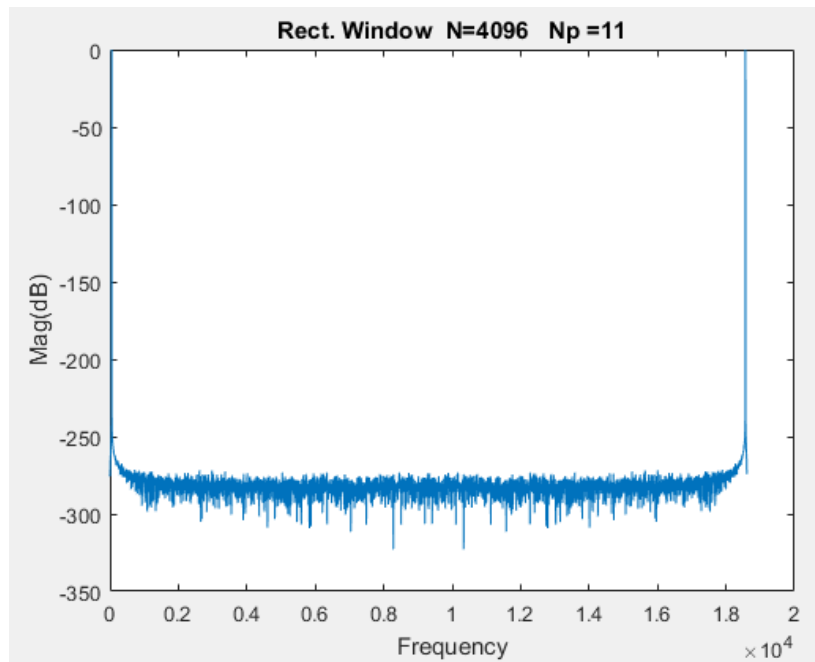
```
Columns 37 through 48
```

```
-265.6690 -265.3833 -264.5215 -264.5995 -268.4617 -266.2074 -265.4619 -265.3134 -267.1584 -267.0717 -265.9300 -268.4336
```

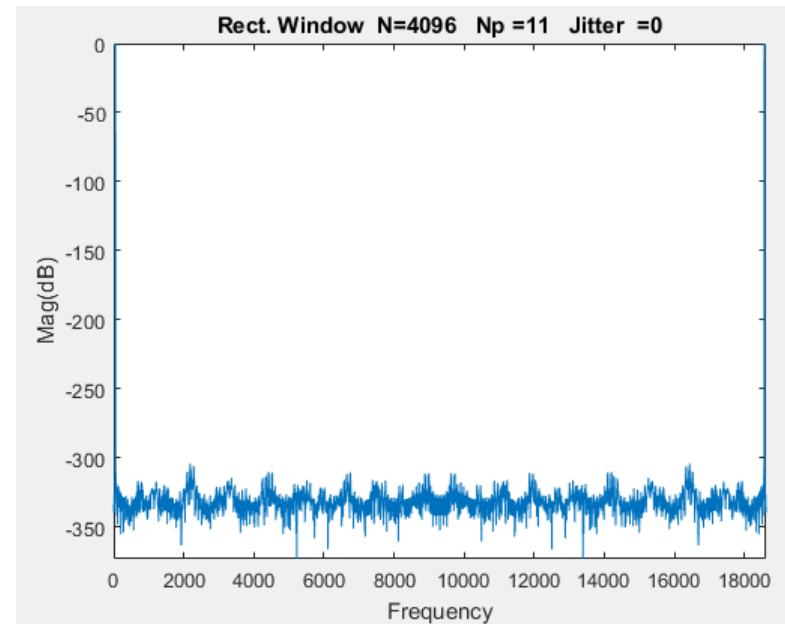
4096 Samples with Strobe Period



Comparison 4096 Samples with Strobe Period Sweep

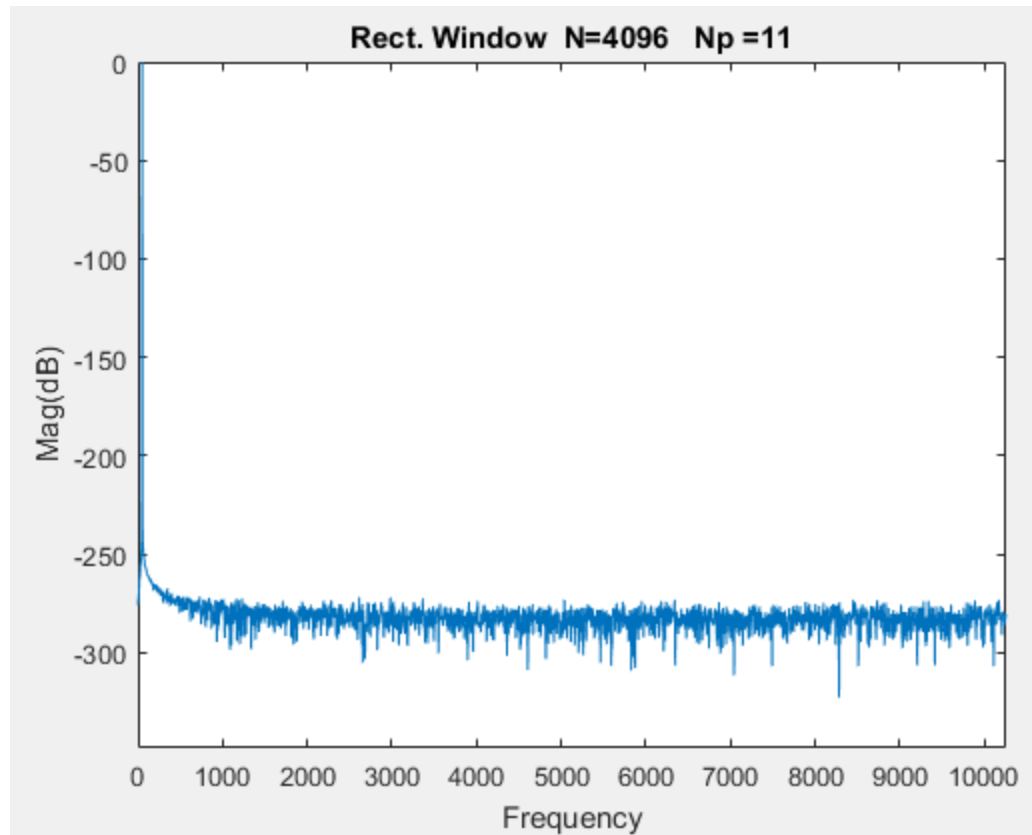


Spectre

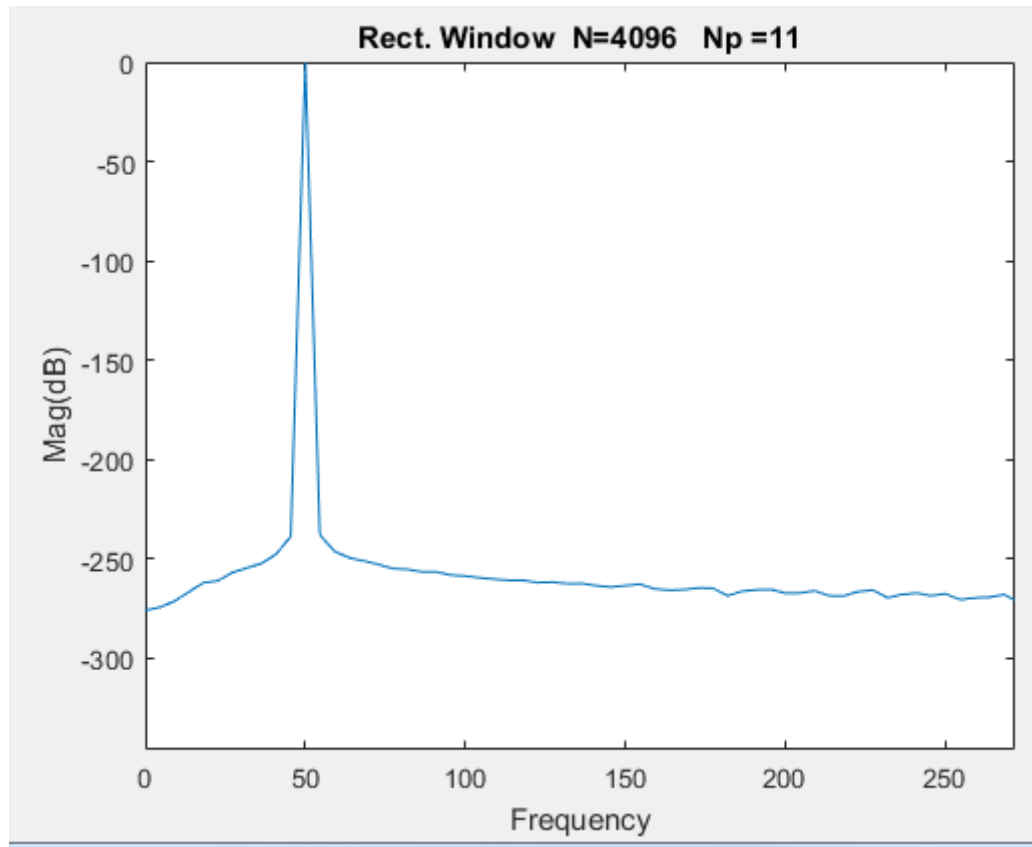


MatLab

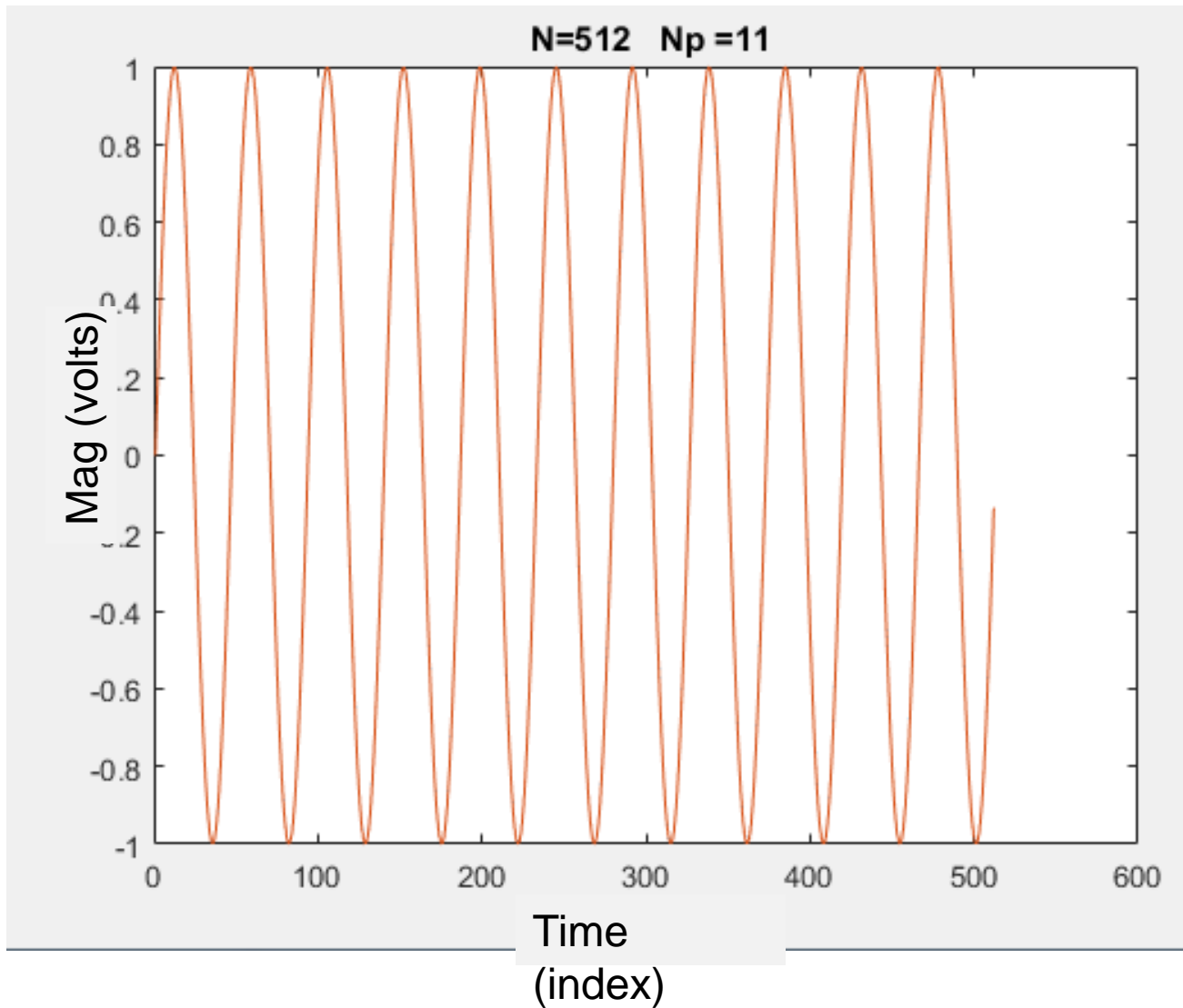
4096 Samples with Strobe Period



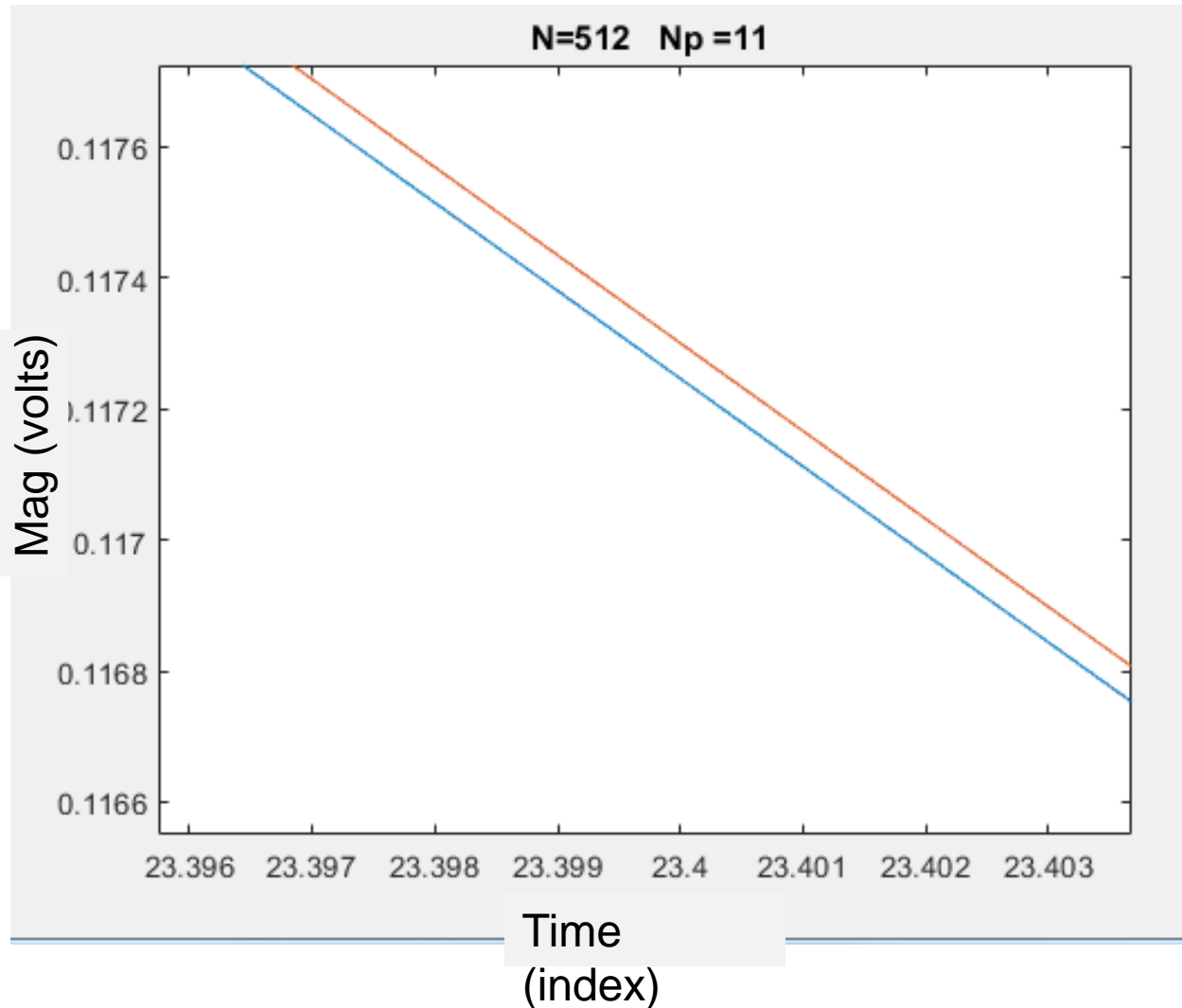
4096 Samples with Strobe Period



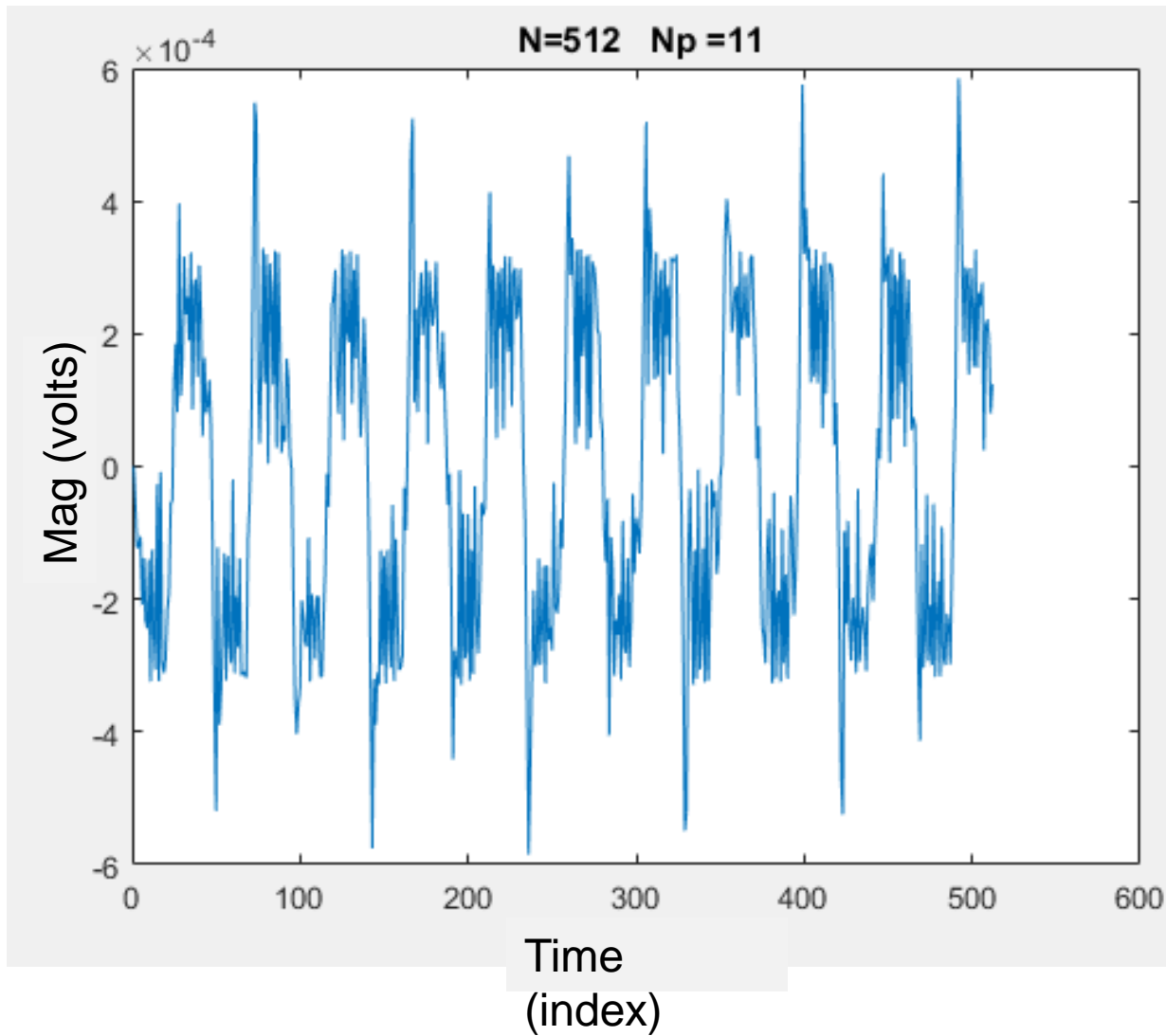
Superimposed Standard/Strobe Sweep



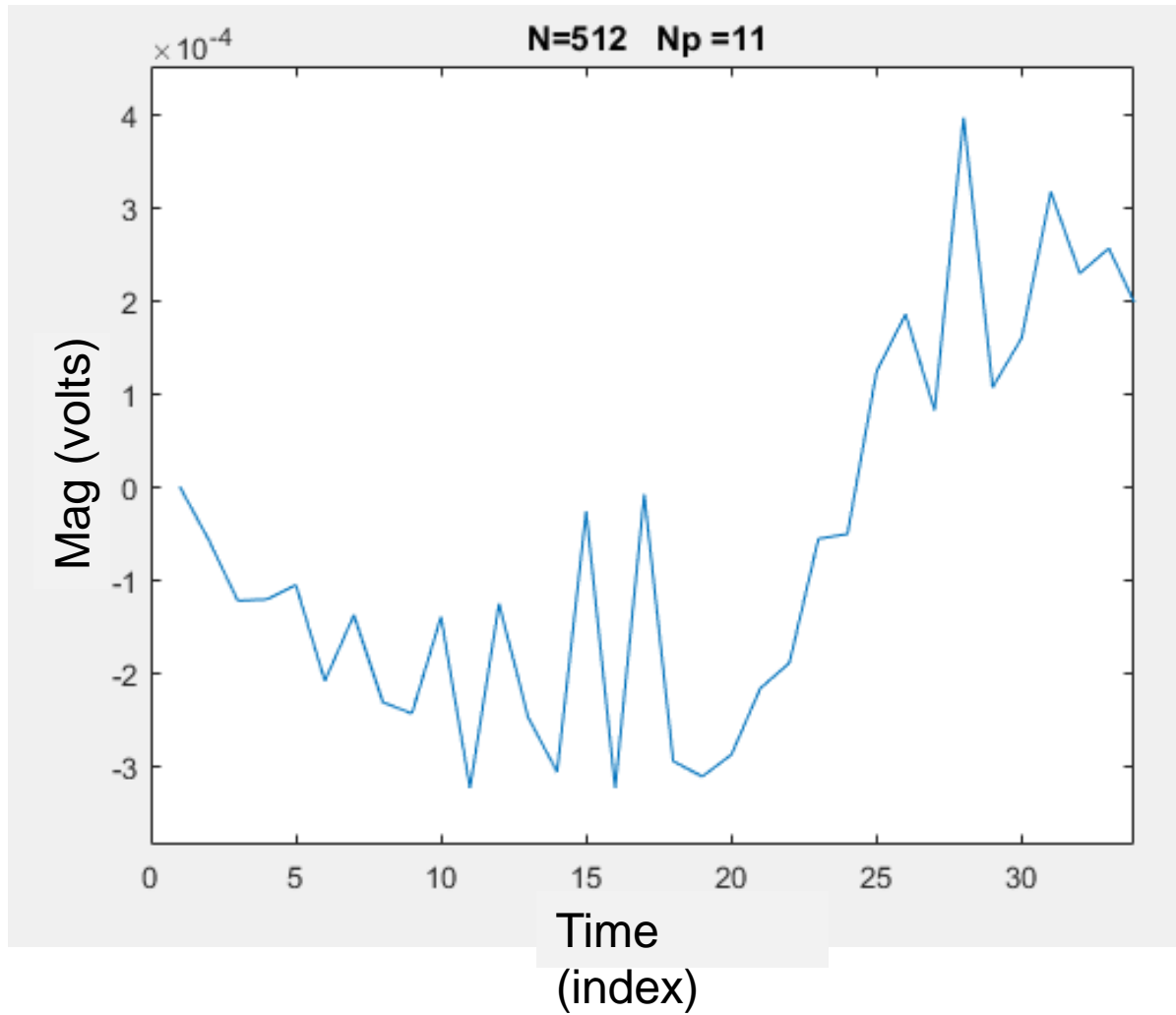
Superimposed Standard/Strobe Sweep



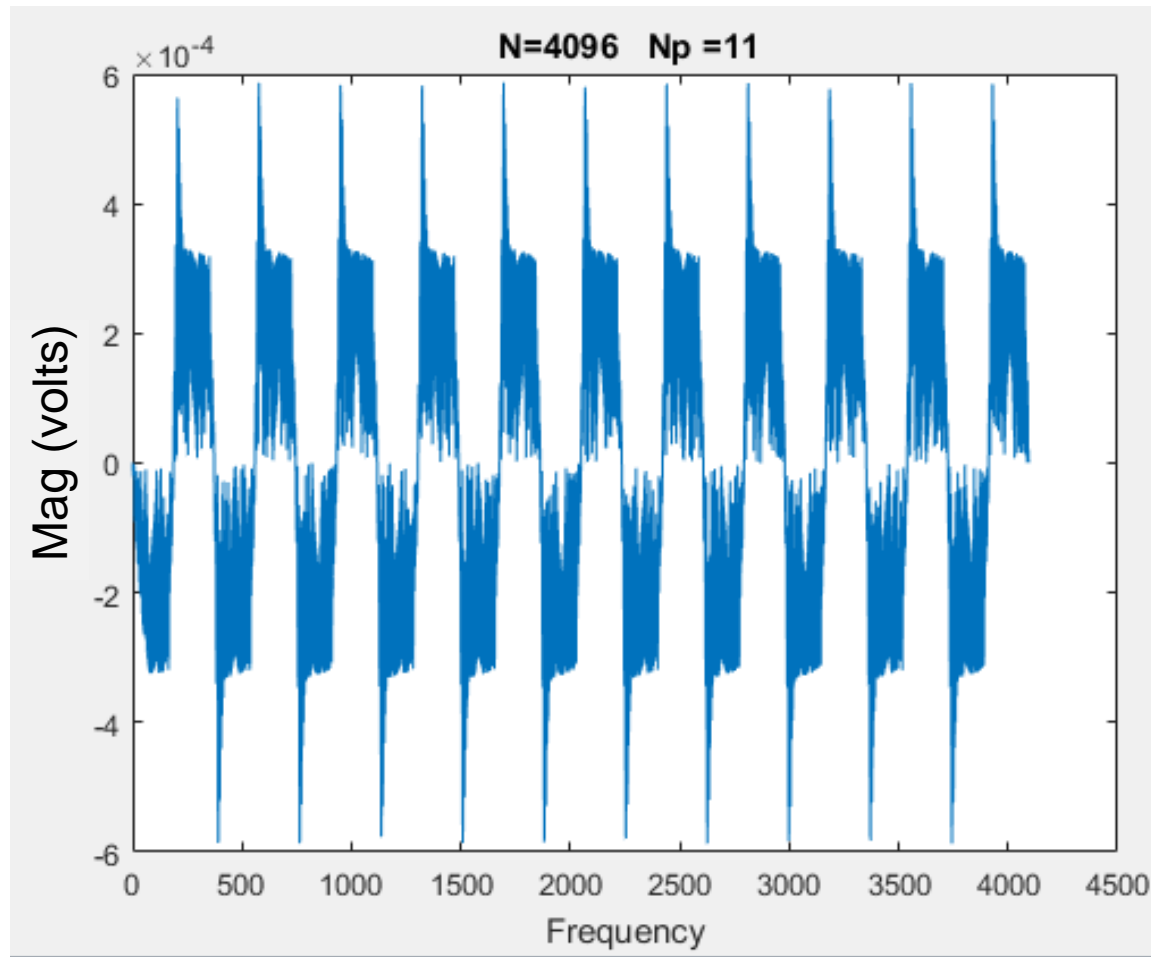
Difference Standard/Strobe Sweep



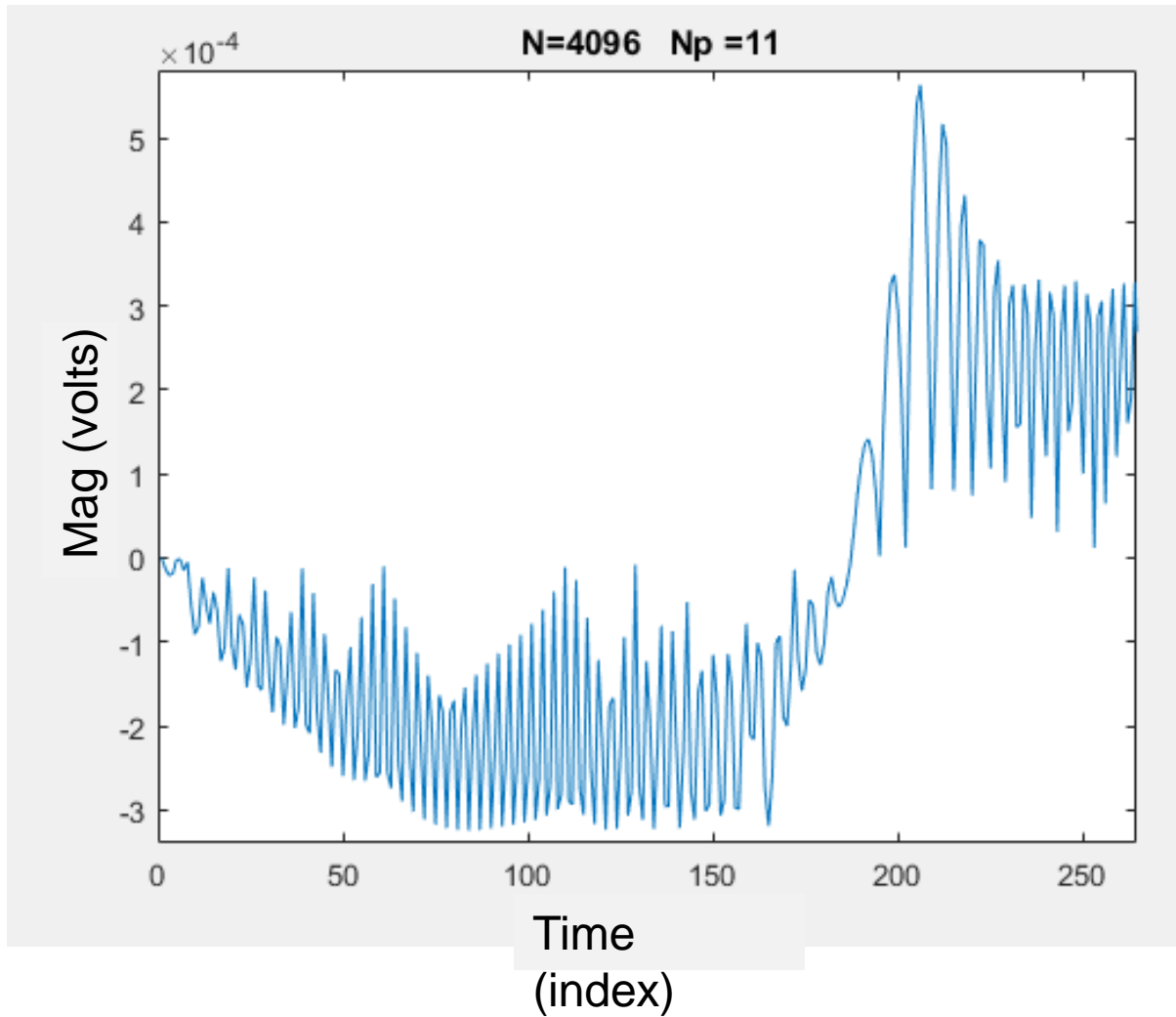
Difference Standard/Strobe Sweep



Difference Standard/Strobe Sweep



Difference Standard/Strobe Sweep



Addressing Spectral Analysis Challenges

- Problem Awareness
- Windowing and Filtering
- Post-processing

Problem Awareness

THEOREM: Consider a periodic signal with period $T=1/f$ and sampling period $T_s=1/f_s$. If N_p is an integer, $x(t)$ is band limited to f_{MAX} , and $f_s > 2f_{max}$, then

$$|A_m| = \frac{2}{N} |X(mN_p + 1)| \quad 0 \leq m \leq h - 1$$

and $X(k) = 0$ for all k not defined above

where $\langle X(k) \rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_s) \rangle_{k=0}^{N-1}$

N =number of samples, N_p is the number of periods, and $h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_p}\right)$

- Hypothesis is critical
- Even minor violation of the premise can have dramatic effects
- Validation of all tools is essential
- Learn what to expect

Filtering - a strategy to address the aliasing problem

- A lowpass filter is often used to enforce the band-limited requirement if not naturally band limited
- Lowpass filter often passive
- Lowpass filter design often not too difficult
- Minimum sampling frequency often termed the Nyquist rate.

End of Lecture 6